SENSITIVITY ANALYSIS OF SOLIDIFICATION PROCESS WITH RESPECT TO GRAINS GEOMETRY

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SUMMARY

In the paper the influence of the grains geometry on the course of solidification process is analyzed. In order to solve the problem the parameter sensitivity analysis has been applied. The mathematical model of thermal processes proceeding in the casting domain is constructed on the basis of the micro-macro approach. On the outer surface of the domain considered the Robin condition is assumed. On the stage of numerical simulation the combined variant of the BEM has been used.

Key words: solidification, numerical modelling, parameter sensitivity analysis

1. FORMULATION OF THE PROBLEM

The thermal processes proceeding in the casting domain are determined by the Fourier equation in the form

$$ P(x) \in \Omega : \quad c(T) \frac{\partial T(x, t)}{\partial t} = \nabla \left[ \lambda(T) \nabla T(x, t) \right] + L_v \frac{\partial f_S(x, t)}{\partial t} \quad (1) $$

where $T(x, t)$ is the casting temperature, $f_S(x, t)$ is the solid state fraction at the point $x$, $c(T)$ is the volumetric specific heat, $\lambda(T)$ is the thermal conductivity, $L_v$ is the volumetric latent heat, $x, t$ denote the spatial co-ordinates and time. Now, the problem of source function will be discussed.

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A temporary value of solid fraction $f_S$ of the metal at point considered is given by the Johnson–Mehl–Avrami–Kolmogorov type equation [1, 2]

$$f_S = 1 - \exp(-\omega)$$  \hspace{1cm} (2)

where

$$\omega = \omega(x, t) = \frac{4}{3} \pi \nu \frac{\partial N}{\partial t} \left[ \int u \, d\tau \right]$$  \hspace{1cm} (3)

In equation (3) $N$ is a number of nuclei (more precisely: density [nuclei/m$^3$]), $\nu$ is the coefficient equal to 1 for the spherical grains and $\nu<1$ for dendritic growth, $u$ is a rate of solid phase growth, $\tau$ is a moment of crystallization process beginning. If we assume the constant number of nuclei then

$$\omega = \frac{4}{3} \pi \nu N \left[ \int u \, d\tau \right]$$  \hspace{1cm} (4)

The solid phase growth is determined by equation

$$u = \frac{\partial R}{\partial t} = \mu \Delta T^p$$  \hspace{1cm} (5)

where $R$ is a grain radius, $\mu$ is a growth coefficient, $p$ is the exponent from the interval $p \in [1, 2]$, and

$$\Delta T = \Delta T(x, t) = T^* - T(x, t)$$  \hspace{1cm} (6)

is the undercooling below the solidification point $T^*$. Additionally we assume that for $T > T^*$: $\Delta T=0$, in other words $u=0$ and then the lower limit in integrals (3) and (4) equals $t=0$. So

$$\omega = \frac{4}{3} \pi \nu N \left[ \int_0^{\mu \Delta T^p} d\tau \right]$$  \hspace{1cm} (7)

Introducing (2) to the equation (1) one obtains
\[ c(T) \frac{\partial T(x,t)}{\partial t} = \nabla \left[ \lambda(T) \nabla T(x,t) \right] + L_c \exp(-\omega) \frac{\partial \omega}{\partial t} \]  

(8)

and the source function is given by the formula

\[ Q(x,t) = L_c \exp(-\omega) \frac{\partial \omega}{\partial t} = 4 \pi v NL_\mu \Delta T^\nu \left( \int_0^\Delta T^\nu d\tau \right)^3 \exp \left[ -\frac{4}{3} \pi v N \int_0^\Delta T^\nu d\tau \right] \]  

(9)

On the outer surface of the casting the Robin condition is assumed

\[ P(x) \in \Gamma: \ -\lambda(T) \frac{\partial T(x,t)}{\partial n} = \alpha \left[ T(x,t) - T_a \right] \]  

(10)

where \( \alpha \) is the substitute heat transfer coefficient between casting and environment, \( T_a \) is the ambient temperature, \( \partial n \) is the normal derivative. The initial condition in the form

\[ t = 0: \ T(x,0) = T_0 \]  

(11)

is also given.

As was mentioned, the influence of the grains geometry on the value of exponent \( \omega \) is determined by coefficient \( v \). The physical interpretation of this coefficient is shown in Figure 1.

In the next part of the paper the mutual connections between the parameter \( \mu \) and the course of solidification will be discussed and in order to solve the problem the sensitivity analysis will be used.

2. SENSITIVITY ANALYSIS WITH RESPECT TO PARAMETER \( \nu \)

We consider the process described by equation (8) in which the constant values of specific heat and thermal conductivity are assumed. Equation (8) is supplemented by the conditions (10) and (11). In order to analyze the influence of \( \nu \) on the course of solidification the energy equation and boundary-initial conditions should be differentiated with respect to parameter \( \nu \) (the direct approach is applied here [3]).
So, the differentiation of energy equation gives

\[ P(x) \in \Omega: \quad c \frac{\partial U(x,t)}{\partial t} = \lambda \nabla^2 U(x,t) + Q_U(x,t) \]  \hfill (12)

where \( U(x,t) = \partial T(x,t)/\partial \nu \), \( Q_U(x,t) = \partial Q(x,t)/\partial \nu \), at the same time the function \( Q_U(x,t) \) is equal to

\[ Q_U(x,t) = 4\pi NL_{\nu} \mu r \exp \left( -\frac{4}{3} \pi N v r^3 \right). \]  \hfill (13)

\[ \Delta T^p \left( r - 2\nu r_{ij} - \frac{4}{3} \pi N v r^4 + 4\pi N v^2 r^2 r_{ij} - \nu p \Delta T^{p-1}(x,t)r \right) \]

where

\[ r = r(x,t) = \int_0^t \mu \Delta T^p(x,\tau) d\tau \]  \hfill (14)

and

\[ r_{ij} = r_{ij}(x,t) = \int_0^t \mu p \Delta T^{p-1}(x,\tau)U(x,\tau) d\tau \]  \hfill (15)

Now, the boundary and initial conditions are differentiated with respect to \( \nu \) and then

\[ P(x) \in \Gamma: \quad -\lambda \frac{\partial U(x,t)}{\partial n} = \alpha U(x,t) \]  \hfill (16)

whereas

\[ t = 0: \quad U(x,0) = 0 \]  \hfill (17)

3. THE RESULTS OF COMPUTATIONS

In order to solve the basic problem and additional one connected with the sensitivity analysis, the combined variant of the boundary element method has been used [4]. Generally speaking, the method constitutes a certain combination of the BEM and FDM. On the basis of the knowledge of searched function (\( T \) or \( U \)) for time \( t^{f-1} \) the discrete set of this function values for time \( t^{f}=t^{f-1}+\Delta t \) can be found.
In the paper the 1D problem has been solved. The aluminium plate of thickness $L=0.01$ [m] has been considered. The following input data are assumed: thermal conductivity $\lambda=150$ [W/mK], volumetric specific heat $c=2.875$ [MJ/m$^3$K], latent heat per unit of volume $L_v=975$ [MJ/m$^3$], solidification point $T^* = 660^\circ$C, exponent $p=2$, number of nuclei $N=10^{10}$ [nucl/m$^3$], growth coefficient $\mu=3 \times 10^{-6}$ [m/sK$^2$], initial temperature $T_0=670^\circ$C, heat transfer coefficient $\alpha=100$ [W/m$^2$K], ambient temperature $T_a=30^\circ$C, shape coefficient $\nu=0.8.$

In Figure 2 the cooling curves at the axis of symmetry and the boundary are shown. In particular, the region of recalescence effect is presented. The successive curves correspond to the basic solution (central lines) and the solutions obtained on the basis of sensitivity analysis under the assumption that $\Delta\nu=\pm 0.2$. It should be pointed out that value $\nu=1$ corresponds to the spherical grains. The possibility of the basic solution transformation results from the Taylor formula

$$T(x,t,\nu \pm \Delta\nu) = T(x,t,\nu) \pm U(x,t)\Delta\nu$$ (18)

Figure 3 shows the changes of temperatures (axis and boundary) due to the change of parameter $\nu$ ($\Delta\nu=\pm 0.2$). The assumed change of shape parameter (the border values determine the spherical grains and dendritic ones) causes the change of the local temperature less than 1K. So, from the numerical point of view the value of $\nu$ is not very essential for the course of thermal processes in the casting domain.

Summing up, the paper presented shows the possibilities of parameter sensitivity analysis in the case of second generation models and the algorithm presented seems to be quite...
effective and exact tool for the investigations from this scope. The generalization of the algorithm on more complex geometrical problems does not cause the essential difficulties.

Fig. 3. Changes of temperature
Rys. 3. Zmiany temperatury

REFERENCES


ANALIZA WRAŻLIWOŚCI PROCESU KRZEPNIĘCIA
ZE WZGLĘDU NA GEOMETRię ZIAREN

STRESZCZENIE


Recenzował Prof. Bohdan Mochnacki