MODEL OF CAST IRON SOLIDIFICATION
USING THE ARTIFICIAL MUSHY ZONE APPROACH

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SUMMARY

The solidification of typical cast iron proceeds partially in an interval of temperature and partially at a constant temperature. In order to apply the simple and effective approach called the one domain method the substitute thermal capacity of material considered must be introduced. It is possible under the assumption that the temperature $T_e$ corresponding to the eutectic change will be substituted by a change proceeding in a certain interval $[T_e - \Delta T, T_e + \Delta T]$. In this way the subdomain of artificial mushy zone is introduced.

The value of local substitute thermal capacity is defined according to concept presented by Hsiao [5] and modified by Lara et al [6]. In the paper the theoretical aspects of problem and also the example of numerical simulation are discussed.

Key words: cast iron solidification, artificial mushy zone, numerical methods

1. INTRODUCTION

The concept of artificial mushy zone definition was presented in the past repeatedly [1, 2, 3, 4]. The discontinuity of enthalpy at point $T_e$ is substituted by a continuous, for instance, linear function. Next, the volumetric specific heat of casting domain is defined. If the one domain approach is used then the thermal processes proceeding in the casting domain are described by the following equation

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\[
\left[ c(T) - L \frac{df_s}{dT} \right] \frac{\partial T(x, t)}{\partial t} = \text{div} \left[ \lambda(T) \, \text{grad} \, T(x, t) \right]
\]  

(1)

where \( c, \lambda \) are a volumetric specific heat and thermal conductivity, \( L \) is a volumetric latent heat, \( f_s \) is the solid state fraction at the neighborhood of considered point from casting domain, \( T, x, t \) denote temperature, spatial co-ordinates and time. The parameter

\[
C(T) = c(T) - L \frac{df_s}{dT}
\]

(2)

is called a substitute thermal capacity of material and finally one obtains the energy equation in the form.

\[
C(T) \frac{\partial T(x, t)}{\partial t} = \text{div} \left[ \lambda(T) \, \text{grad} \, T(x, t) \right]
\]

(3)

The mathematical model of thermal processes proceeding in a casting - mould system contains additionally the energy equation for a mould sub-domain, namely

\[
c_m(T) \frac{\partial T_m(x, t)}{\partial t} = \text{div} \left[ \lambda_m(T) \, \text{grad} \, T_m(x, t) \right]
\]

(4)

where \( c_m, \lambda_m \) are a mould volumetric specific heat and a mould thermal conductivity.

The boundary conditions for an outer surface of a system and a contact one are also known [1, 2]. The initial temperature distributions in a system considered are given, too.

2. ENTHALPY FUNCTION

The substitute thermal capacity can be, generally speaking, defined as a derivative of material physical enthalpy \( H \) with respect to temperature

\[
C(T) = \frac{dH(T)}{dT}
\]

(5)

So, the course of function determining the cast iron substitute thermal capacity can be found using the enthalpy - carbon diagram presented among others in [1, 2]. On the basis of this diagram the dependence between temperature and enthalpy for the assumed concentration of alloy component can be determined (see: [3, 4]). As an example the course of function \( H = H(T) \) for \( c = 3\% \) is shown in Figure 2 [4] (the real course of
\[ H(T) \] has been approximated by the broken line.

In order to define the substitute thermal capacity of material considered the fragment corresponding to 'a leap' of enthalpy at the temperature corresponding to eutectic change must be substituted by a differentiable function, e.g. linear one. So, the temperature \( T_e \) should be 'extended' on a certain interval \( [T_e - \Delta T, T_e + \Delta T] \) and then one obtains the modified course of \( H = H(T) \), as in Figure 3 [4].

The function \( C(T) \) found on the basis of formula (5) for enthalpy function presented in Figure 3 is a staircase one. In order to improve the results of numerical simulation it is
possible to introduce the definition of substitute thermal capacity in direction $k$ [5, 7].

Let us denote by 0 and $k$ two adjoining nodes resulting from the casting domain discretization. The thermal capacity $C_{0k}$ is equal to

$$C_{0k} = \frac{1}{T_k - T_0} \int_{T_0}^{T_k} C(T) \, dT$$

(6)

Next one defines the substitute thermal capacity corresponding to the node considered in the following way

$$C_0 = \sum_{k=1}^{N} C_{0k} \, w_k$$

(7)

where $w_k$ are the tapering functions determined by a distance between central node 0 and adjoining node $k$. In the case of regular rectangular mesh typical for FDM the tapering functions equal $1/2$ (1D problem), $1/4$ (2D) or $1/6$ (3D).

The algorithm of $C_{0k}$ computations for staircase function was presented by Hsiao [5], but his proposition is rather complex (from the numerical point of view). The other way was discussed by Lara et.al. in [6] and this approach is applied here.

One can see that

$$\int_{T_0}^{T_k} C(T) \, dT = \int_{T_0}^{T_k} C(T) \, dT - \int_{T_0}^{T_0} C(T) \, dT$$

(8)

and

$$\int_{T_0}^{T_k} C(T) \, dT - \int_{T_0}^{T_0} C(T) \, dT = H(T_k) - H(T_0)$$

(9)

is a difference between enthalpies corresponding to $T_k$ and $T_0$. Next

$$C_{0k} = \frac{H(T_k) - H(T_0)}{T_k - T_0}$$

(10)

If the nodes $x_0$ and $x_k$ belong to the same 'phase' then $C_{0k}$ corresponds to the slope of adequate fragment of broken line, while if the nodes belong to the different 'phase' then $C_{0k}$ corresponds to the slope of chord between points $(T_0, H_0)$ and $(T_k, H_k)$ [6].
3. EXAMPLE OF COMPUTATIONS

We consider the solidification of the cast iron plate \((g = 1.9 \text{ cm})\). The casting is produced in the typical sand mix \((\lambda_m = 0.75 \text{ W/mK}, \ c_m = 1.64 \text{ MJ/m}^3\text{K})\). The thermophysical parameters of the casting material have been taken from [1].

The temperature corresponding to the beginning of solidification equals \(1300 \ ^\circ\text{C}\), eutectic point \(T_e = 1145 \ ^\circ\text{C}\), pouring temperature \(T_p = 1400 \ ^\circ\text{C}\), initial mould temperature \(T_{m0} = 20 \ ^\circ\text{C}\). In order to define the substitute thermal capacity of the eutectic phase change the temperature \(T_e\) has been substituted by the interval \([T_e - 2, T_e + 2]\). The 1D problem has been solved using the explicit variant of FDM [1, 2, 7] and the geometrical mesh with the constant step \(h = 0.001 \text{ m}\) has been introduced. In Figure 4 the temperature distribution in the region of facing sand and the casting domain for times 10, 30, 60, 120, and 180s (1, 2, 3, 4, 5) are shown. The obtained results have been compared with the more complex solution presented in [8] and their conformability is quite satisfactory.

![Temperature profiles](image)

Fig. 4. Temperature profiles.
Rys. 4. Profile temperature.

Summing up it seems that the introduction of substitute thermal capacity in the form presented in this paper can constitute an effective tool for numerical modelling of heat transfer processes proceeding in the solidifying cast iron.

REFERENCES

MODEL KRZEPNIĘCIA ŻELIWA Z WYKORZYSTANIEM SZTUCZNEJ STREFY DWUFAZOWEJ

STRESZCZENIE

Krzepnięcie typowego żeliwa zachodzi częściowo w przedziale temperatury, a częściowo w stałej temperaturze. Jedną z prostych i efektywnych możliwości realizacji obliczeń jest wprowadzenie zastępczej pojemności cieplnej rozważanego materiału. Temperaturę $T_e$ odpowiadającą przemianie eutektycznej zastąpiono pewnym przedziałem temperatury $[T_e - \Delta T, T_e + \Delta T]$. W ten sposób wprowadzono podobszar nazywany sztuczną strefą dwufazową. Wartość lokalnej zastępczej pojemności cieplnej zdefiniowano zgodnie z podejściem prezentowanym przez Hsiao i zmodyfikowanym przez Larę i innych. W artykule przedyskutowano teoretyczne aspekty problemu, a także zaprezentowano przykład obliczeń numerycznych.

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