Generalization of temporary temperature field correction method in numerical modelling of solidification

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Abstract

The temporary temperature field correction method constitutes a very effective tool for numerical modelling of solidification. The most general version of this algorithm has been presented by Mochnacki [1] and Szopa [2]. The basic idea of the method consists in the computations of temperature field for homogeneous domain (e.g. corresponding to molten metal or solid body) and obtained in this way a discrete temperature field for time \( t \) is in adequate way 'rebuilt' using the simple mathematical formulae. In this way the non-homogeneous geometry of solidifying casting is taken into account. This approach can be used in the case of macro modelling, in particular when the one domain approach [3, 4, 5, 6] is applied. The basic assumption of the algorithm presented in [1] and [2] was that the substitute thermal capacity (STC) \([7, 8]\) of casting material can be approximated by a piece-wise constant function. In this paper the generalization of the method on a case of practically optional function describing a course of STC. In the final part of the paper one can find the example of numerical simulations using this approach.

Keywords: Application of Information Technology to the Foundry Industry; Solidification Process; Numerical Techniques; Numerical Simulation of Casting Solidification

1. Introduction

The primary version of method introducing an artificial homogenization of casting domain has been presented by Ruddle [9] (Temperature Recovery Method). It can be used in the case of numerical modelling of pure metals or eutectic alloys solidification (the Stefan problem). Let us assume (it is not necessary) that the thermophysical parameters of liquid and solidified part are constant and equal. It is a certain simplification, of course. The 'reserve' of temperature \( \Theta \) is defined as the quotient of the volumetric latent heat \( L \) to \( c \) (volumetric specific heat of metal), this means

\[
\Theta = \frac{L}{c}
\]  

(1)

If the node \( X_0 \) belongs to the casting domain \( (X_0 \in \Omega_d) \) then at the moment \( t = 0 \) the temperature at this point corresponds to the pouring temperature as well as the temperature reserve results from (1). For the typical cast steel this quantity is of order \( 300 \mathrm{K} \).

On the basis of the optional numerical method we find a discrete temperature field at the set of points \( X_0 \in \Omega_d \) for successive levels of time. If during the interval \( \Delta t = t^{i+1} - t^i \) the temperature \( T_{d}^{i+1} \) at point \( X_0 \) decreases below the solidification point \( T^* \) then it is assumed that the temperature at this point is equal to \( T^* \) and the reserve of temperature must be decreased, namely...
\[ \Theta_0^{i+1} = \Theta_0 - \Delta \Theta_i^{i+1}, \] where \( \Delta \Theta_i^{i+1} = T^* - T_0^{i+1} \). So, the temperature field obtained at time \( t^{i+1} \) is corrected in following way:

i. For the nodes in which \( T_0^{i+1} > T^* \), the temperature reserve \( \Theta_0 \) is untouched and equal to its initial value. The calculated temperature \( T_0^{i+1} \) is, of course, accepted.

ii. For the nodes in which \( T_0^{i+1} = T^* \) and \( T_0^{i+1} < T^* \) it is assumed that \( T_0^{i+1} = T^* \) and the TRM procedure is initiated.

iii. For the nodes in which \( T_0^{i+1} = T^* \), \( T_0^{i+1} < T^* \), \( \Theta_0^{i+1} > 0 \) it is assumed that \( T_0^{i+1} = T^* \) and the temperature reserve is decreased according the formula \( \Theta_0^{i+1} = \Theta_0 - (T^* - T_0^{i+1}) \).

iv. For the nodes in which \( T_0^{i+1} < T^* \) and \( \Theta_0^{i+1} \leq 0 \) the obtained value of temperature is accepted.

Corrected in this way temperature field in \( \Omega_0 \) illustrates the thermal state in casting domain at the moment \( t^{i+1} \), as well as this constitutes a pseudo-initial condition for the next step of computations.

In Figure 1 the diagram enthalpy - temperature for the case discussed and additionally the idea of method discussed are shown. The variant of TRM concerning the typical binary alloys [10] solidification was presented, among others, by Majchrzak [11].

Let us assume that the temperature at node \( X_0 \) decreases below \( T_L \) (liquidus border temperature), in other words the node ‘walking’ to mushy zone region. The next computations are realized assuming that \( X_0 \) belongs as before to the molten metal, but the local temperature is corrected using the formula corresponding to Figure 2. In this Figure \( T \) denotes the temperature found under the assumption that node \( X_0 \) belongs to the liquid phase, \( U \) denotes the corrected value.

The same way of correction can be used for nodes belonging to solidified part of casting (Figure 2). This method of temperature recalculation is proper when one can assume that the thermal conduc-

\[ \text{tivity of metal can be treated as a constant value. The mathematical proof of this limitation was presented in [1].} \]

The generalization of algorithm above discussed was presented in [1, 2] and this approach was successfully used as the procedure supplementing the boundary element method [2, 11] for parabolic equations (transient heat diffusion). An interval \( [T^*, T_0] \) (ambient temperature - initial temperature) has been divided into sub-intervals, for which one can assume the constant value of volumetric specific heat (finally \( c(T) \) is approximated by a piece-wise constant function). The limits of sub-interval \( [U_m, U_m] \) correspond to the ‘phase’ \( m \) - Figure 3.
volumetric specific heat is a minimal one. Let us assume that the nodal temperature at point \( X_0 \) correspond to phase \( m \) - Figure 4.

Denoting

\[
\Delta_m = c_m \left[ T(X_0, t) - U_{m1} \right]
\]

\[
\Delta_{m1} = \Delta_m + c_{m1} \left[ U_{m2} - U_{m3} \right]
\]

(2)

\[
\Delta_{m2} = \Delta_{m1} + c_{m2} \left[ U_{m4} - U_{m5} \right]
\]

etc.

where \( U_m, U_{m1}, U_{m2}, \ldots, U_{m5} \) are the temperatures limiting the successive 'phases'. The physical interpretation of \( \Delta \) is self-evident, they are the changes of unitary enthalpy \([\text{J/m}^3]\) corresponding to temperature changes shown in Figure 4.

The procedure of temperature correction at point \( X_0 \) is the following.

If

\[
c_0 \left[ T(X_0, t) - T(X_0, t + \Delta t) \right] < \Delta_m
\]

(3)

where \( c_0 \) is a 'basic' specific heat, then the corrected value \( T(X_0, t + \Delta t) \) results from equation

\[
\hat{T}(X_0, t + \Delta t) = T(X_0, t) - \frac{c_0}{c_m} \left[ T(X_0, t) - T(X_0, t + \Delta t) \right]
\]

(4)

\[\text{Fig. 4. Values } \Delta T_m \text{ and the next}\]

If

\[
\Delta_m < c_0 \left[ T(X_0, t) - T(X_0, t + \Delta t) \right] < \Delta_{m1}
\]

(5)

then

\[
c_0 \left[ T(X_0, t) - T(X_0, t + \Delta t) \right] = \Delta_m + c_{m1} \left[ U_{m1} - \hat{T}(X_0, t + \Delta t) \right]
\]

(6)

\[
\hat{T}(X_0, t + \Delta t) = \frac{U_{m1} + c_{m1} \left[ T(X_0, t) - T(X_0, t + \Delta t) \right]}{c_{m1}}
\]

(7)

If

\[
\Delta_{m1} < c_0 \left[ T(X_0, t) - T(X_0, t + \Delta t) \right] < \Delta_{m2}
\]

then

\[
c_0 \left[ T(X_0, t) - T(X_0, t + \Delta t) \right] = \Delta_{m1} + c_{m2} \left[ U_{m2} - \hat{T}(X_0, t + \Delta t) \right]
\]

(9)

and finally

\[
\hat{T}(X_0, t + \Delta t) = \frac{U_{m2} + c_{m2} \left[ T(X_0, t) - T(X_0, t + \Delta t) \right]}{c_{m2}}
\]

(10)

The similar formulas can be found for the others transitions.

2. Generalized temperature field correction method

Let us assume that the volumetric specific heat (substitute thermal capacity) of casting material is an optional function of temperature \( c(T) \), while the thermal conductivity is a constant value. We denote by \( T(X_0, t) \) and \( T(X_0, t + \Delta t) \) the temperatures at point \( X_0 \) for two successive time levels. The computations are realized under the assumption that the volumetric specific heat of material correspond to basic value \( c_0 \). The change of physical enthalpy of a control volume \( V_0 \) for which the point \( X_0 \) is a central one equals

\[
\Delta H_0 = c_0 \left[ T(X_0, t + \Delta t) - T(X_0, t) \right] V_0
\]

(11)

Taking into account the real course of \( c(T) \) we should compare the value resulting from (11) with the expression

\[
\frac{\tau(x_{o1} + \Delta t)}{\tau(x_{o1})} \int_{x(x_{o1})} c(\mu) \, d\mu
\]

(12)

Finally the upper limit of integral in equation

\[
c_0 \left[ T(X_0, t + \Delta t) - T(X_0, t) \right] = \int_{x(x_{o1})} c(\mu) \, d\mu
\]

must be found. It is quite simple problem if one uses the methods of numerical integration.
3. Example of computations

The example of numerical computations concerns the simulation of heat treatment of spherical \( R = 30 \text{mm} \) steel casting for which the volumetric specific heat is approximated by the polynomial [12]. The input data have been taken from [2]. The initial temperature of casting: \( T_i = 900 \degree \text{C} \), cooling water temperature: \( T_w = 30 \degree \text{C} \), heat transfer coefficient: \( \alpha = 200 \text{ W/m}^2 \text{K} \), thermal conductivity of material \( \lambda = 35 \text{ W/mK} \). In Figure 5 the cooling curves at the points selected in casting domain are presented.

![Fig. 5. Cooling curves](image)

Summing up, the generalization of temporary temperature field correction method gives the new possibilities of non-steady heat transfer modelling. One can see, that in the case of \( c(T) \) approximation by a piece-wise constant function, the algorithm presented reduces to the previous version of the method.

Streszczenie


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References