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Numerical modelling of solidification process using interval boundary element method

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Abstract

In this paper an application of the interval boundary element method for solving problems with interval thermal parameters and interval source function in a system casting-mould is presented. The task is treated as a boundary-initial problem in which the crystallization model proposed by Mehl-Johnson-Avrami-Kolmogorov has been applied. The numerical solution of the problem discussed has been obtained on the basis of the interval boundary element method (IBEM). The interval Gauss elimination method with the decomposition procedure has been applied to solve the obtained interval system of equations. In the final part of the paper, results of numerical computations are shown.

Keywords: Application of information technology to the foundry industry, Solidification process, Interval arithmetic, Interval boundary element method, Interval source function

1. Introduction

The solidification process proceeding in the volume of pure metal or alloy can be analyzed as a macroscopic one [1, 2, 3] but the microscopic aspects of the phenomena considered, in particular the nucleation and the nuclei growth [4, 5, 6, 7, 8] must be taken into account. The energy equation contains the term called the source function proportional to the solidification rate determined by the changes of nuclei density and temporary grains dimensions. Such approach is widely known.

The interval analysis of the typical tasks of the solidification process seems be more effective because experimental estimations of the grain density, the growth and the thermal parameters are difficult. These values are dependent on technological conditions, object geometry etc. So, it seems natural, that the parameters appearing in the mathematical model of solidification process should be treated as the interval values [9, 10]. This assumption is closer to the real physical conditions of the process considered.

Let us consider the solidification process in heterogeneous domain $\Omega = \Omega_1 \cup \Omega_2$ of the casting and mould (see Figure 1).

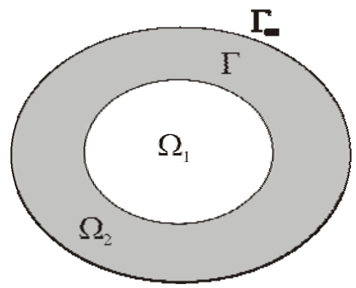

Fig. 1. Domain considered

The heat conduction process in the casting sub-domain Ω_1 is described by the following energy equation

$$x \in \Omega_1: \quad \tilde{c}_1 \frac{\partial T_1(x,t)}{\partial t} = \tilde{\lambda}_1 \nabla^2 T_1(x,t) + \tilde{Q}(x,t) \tag{1}$$

where $\tilde{c}_1$ is the interval volumetric specific heat for the casting sub-domain, $\tilde{\lambda}_1$ is the interval thermal conductivity, $\tilde{Q}(x,t)$ is the interval source function, T_1, x, t denote temperature, spatial co-ordinates and time, respectively.

The temperature field in the mould sub-domain Ω_2 is determined by the energy equation

$$x \in \Omega_2: \quad \tilde{c}_2 \frac{\partial T_2(x,t)}{\partial t} = \tilde{\lambda}_2 \nabla^2 T_2(x,t) \tag{2}$$

The above equations (1) and (2) must be supplemented by the boundary-initial conditions

$$\begin{cases} x \in \Gamma_\infty: & F\left[T_2(x,\ t),\ \dfrac{\partial T_2(x,\ t)}{\partial n}\right] = 0 \\ t = 0: & T_e(x,\ 0) = T_{e0}(x), \quad e = 1, 2 \end{cases} \tag{3}$$

and the continuity condition on the contact surface between the casting and mould

$$x \in \Gamma: \quad \begin{cases} -\tilde{\lambda}_1 \dfrac{\partial T_1(x,\ t)}{\partial n} = -\tilde{\lambda}_2 \dfrac{\partial T_2(x,\ t)}{\partial n} \\ T_1(x,\ t) = T_2(x,\ t) \end{cases} \tag{4}$$

The interval source function $\tilde{Q}(x,t)$ (see eq.1) is depended on the interval volumetric fraction of the solid state $\tilde{S}(x,t)$ at the neighborhood of the point considered x and takes the following form

$$\tilde{Q}(x,t) = Q_{cr} \frac{\partial \tilde{S}(x,t)}{\partial t} = Q_{cr} \frac{\partial}{\partial t}\left\{1 - \exp\left[-\tilde{\omega}(x,t)\right]\right\} \tag{5}$$

where Q_{cr} is the volumetric latent heat, $\tilde{\omega}(x,t)$ is the interval function and for the spherical grains it is defined using the formula

$$\tilde{\omega}(x,t) = \frac{4}{3}\pi \tilde{N}(x,t)\tilde{R}(x,t)^3 \tag{6}$$

where $\tilde{R}(x,t)$ is the temporary interval radius of the single grain, $\tilde{N}(x,t)$ is the interval grain density.

Denoting by $\tilde{u}(x,t)$ the interval solidification rate the temporary interval radius of the single grain can be written as follows

$$\tilde{R}(x,t) = \int_0^t \tilde{u}(x,\tau)\,\mathrm{d}\tau \tag{7}$$

while the interval solidification rate is defined as

$$\tilde{u}(x,t) = \tilde{\mu}\ \Delta T(x,t)^2 \tag{8}$$

where $\tilde{\mu}$ is the interval value of the growth coefficient, $\Delta T(x,t)$ is the undercooling below the solidification point T_{cr} [6, 9].

Taking into account the interval value of the growth coefficient $\tilde{\mu} = \left\langle \underline{\mu}, \overline{\mu} \right\rangle$ and the constant interval value of the grain density, $\tilde{N} = \left\langle \underline{N}, \overline{N} \right\rangle$ the interval source function $\tilde{Q}(x,t)$ can be expressed as follows

$$\tilde{Q}(x,t) = 4\pi Q_{cr} \tilde{N} \tilde{R}(x,t)^2 \tilde{\mu} \Delta T(x,t)^2 \exp\left[-\frac{4}{3}\pi \tilde{N} \tilde{R}(x,t)^3\right] \tag{9}$$

The interval source function $\tilde{Q}(x,t)$ for the constant interval value of the grain density $\tilde{N} = \left\langle \underline{N}, \overline{N} \right\rangle$ is defined using the following formula

$$\tilde{Q}(x,t) = 4\pi Q_{cr} \tilde{N} \tilde{R}(x,t)^2 \tilde{\mu} \Delta T(x,t)^2 \exp\left[-\frac{4}{3}\pi \tilde{N} \tilde{R}(x,t)^3\right] \tag{10}$$

This interval source function has to be calculated according to the rules of the interval arithmetic [9].

2. Interval arithmetic

Let us consider an interval $\tilde{x}$, which can be defined as a set of the following form [11]

$$\tilde{x} \equiv \left\langle \underline{x}, \overline{x} \right\rangle := \left\{ x \in \boldsymbol{R} \mid \underline{x} \le x \le \overline{x} \right\} \tag{11}$$

where $\underline{x}$ and $\overline{x}$ denote the lower and the upper bounds, respectively. An interval is called thin if $\underline{x} = \overline{x}$ and thick if $\underline{x} < \overline{x}$.

The sum of two intervals $\tilde{a} = \left\langle \underline{a}, \overline{a} \right\rangle$ and $\tilde{b} = \left\langle \underline{b}, \overline{b} \right\rangle$ can be written as

$$\tilde{c} = \tilde{a} + \tilde{b} = \left\langle \underline{a} + \underline{b},\ \overline{a} + \overline{b} \right\rangle \tag{12}$$

The difference is of the form

$$\tilde{c} = \tilde{a} - \tilde{b} = \left\langle \underline{a} - \overline{b},\ \overline{a} - \underline{b} \right\rangle \tag{13}$$

The product of the intervals is described by the following formula

$$\tilde{c}=\tilde{a}\cdot\tilde{b}=\left\langle\begin{array}{l}\min\left(\underline{a}\cdot\underline{b},\,\underline{a}\cdot\overline{b},\,\overline{a}\cdot\underline{b},\,\overline{a}\cdot\overline{b}\right),\\ \max\left(\underline{a}\cdot\underline{b},\,\underline{a}\cdot\overline{b},\,\overline{a}\cdot\underline{b},\,\overline{a}\cdot\overline{b}\right)\end{array}\right\rangle \tag{14}$$

The inversion of the interval $\tilde{b}$ can be written as

$$\tilde{c}=1/\tilde{b}=\left\langle 1/\overline{b},\,1/\underline{b}\right\rangle,\quad 0\notin\left\langle\underline{b},\,\overline{b}\right\rangle \tag{15}$$

The quotient of two intervals can be expressed as

$$\tilde{c}=\tilde{a}/\tilde{b}=\tilde{a}\cdot 1/\tilde{b},\quad 0\notin\left\langle\underline{b},\,\overline{b}\right\rangle \tag{16}$$

3. Interval boundary element method

For simplification a 1D problem has been analysed. The heat conduction process in this case is described by the following energy equations

$$x\in\Omega_e:\quad \tilde{c}_e\frac{\partial T_e(x,t)}{\partial t}=\tilde{\lambda}_e\frac{\partial^2 T_e(x,t)}{\partial x^2}+\tilde{Q}_e(x,t),\quad e=1,2 \tag{17}$$

with the boundary-initial conditions of the form

$$\begin{cases} x=0: & q_1(x,\ t)=q_L\\ x=L_2: & q_2(x,\ t)=q_R\\ x=L_1: & \begin{cases} -\tilde{\lambda}_1\dfrac{\partial T_1(x,\ t)}{\partial n}=-\tilde{\lambda}_2\dfrac{\partial T_2(x,\ t)}{\partial n}\\ T_1(x,\ t)=T_2(x,\ t)\end{cases}\\ t=0: & T_e(x,\ 0)=T_{e0}(x)\end{cases} \tag{18}$$

where $e=1,2$ identify the casting and mould sub-domains. The source function for the mould sub-domain is equal to zero ($\tilde{Q}_2(x,t)=0$).

In this paper the 1st scheme of the boundary element method is used [12, 13, 14, 15]. As first, the time grid must be introduced

$$0=t^0<t^1<t^2<\ldots<t^{f-1}<t^f<\ldots<t^F<\infty \tag{19}$$

with a constant time step $\Delta t=t^f-t^{f-1}$.

Let us consider the constant elements with respect to time

$$t\in\left\langle t^{f-1},\,t^f\right\rangle:\ \begin{cases}\tilde{T}_e(x,t)=\tilde{T}_e(x,t^f)\\ \tilde{q}_e(x,t)=\tilde{q}_e(x,t^f)\end{cases} \tag{20}$$

The boundary integral equation corresponding to the transition $t^{f-1}\to t^f$ for the casting sub-domain is following

$$\begin{aligned}&\tilde{T}_1\left(\xi,t^f\right)+\left[\frac{1}{\tilde{c}_1}\tilde{q}_1\left(x,t^f\right)\int_{t^{f-1}}^{t^f}\tilde{T}_1^*\left(\xi,x,t^f,t\right)dt\right]_{x=0}^{x=L_1}=\\ &\left[\frac{1}{\tilde{c}_1}\tilde{T}_1\left(x,t^f\right)\int_{t^{f-1}}^{t^f}\tilde{q}_1^*\left(\xi,x,t^f,t\right)dt\right]_{x=0}^{x=L_1}+\\ &\int_0^{L_1}\tilde{T}_1^*\left(\xi,x,t^f,t^{f-1}\right)\tilde{T}_1\left(x,t^{f-1}\right)dx+\\ &\frac{1}{\tilde{c}_1}\int_0^{L_1}\tilde{Q}_1\left(x,t^{f-1}\right)\left[\int_{t^{f-1}}^{t^f}\tilde{T}_1^*\left(\xi,x,t^f,t\right)dt\right]dx\end{aligned} \tag{21}$$

where ξ is the observation point, $\tilde{q}_1(x,t^f)$ is the interval heat flux. The interval fundamental solution $T_1^*(\xi,x,t^f,t)$ has the form [11, 16]

$$\tilde{T}_1^*(\xi,x,t^f,t)=\frac{1}{2\sqrt{\pi\tilde{a}_1(t^f-t)}}\exp\left[-\frac{(x-\xi)^2}{4\tilde{a}_1(t^f-t)}\right] \tag{22}$$

where $\tilde{a}_1=\tilde{\lambda}_1/\tilde{c}_1$ is the interval value of the diffusion coefficient. The heat flux resulting from the fundamental solution should be found in analytic way and then

$$\begin{aligned}&\tilde{q}_1^*(\xi,x,t^f,t)=-\tilde{\lambda}_1\frac{\partial\tilde{T}_1^*(\xi,x,t^f,t)}{\partial n}=\\ &\frac{\tilde{\lambda}_1(x-\xi)}{2\sqrt{\pi}\left[\tilde{a}_1(t^f-t)\right]^{3/2}}\exp\left[-\frac{(x-\xi)^2}{4\tilde{a}_1(t^f-t)}\right]\end{aligned} \tag{23}$$

The boundary integral equation corresponding to the transition $t^{f-1}\to t^f$ for the mold sub-domain can be expressed as follows

$$\begin{aligned}&\tilde{T}_2\left(\xi,t^f\right)+\left[\frac{1}{\tilde{c}_2}\tilde{q}_2\left(x,t^f\right)\int_{t^{f-1}}^{t^f}\tilde{T}_2^*\left(\xi,x,t^f,t\right)dt\right]_{x=0}^{x=L_1}=\\ &\left[\frac{1}{\tilde{c}_2}\tilde{T}_2\left(x,t^f\right)\int_{t^{f-1}}^{t^f}\tilde{q}_2^*\left(\xi,x,t^f,t\right)dt\right]_{x=0}^{x=L_1}+\\ &\int_0^{L_1}\tilde{T}_2^*\left(\xi,x,t^f,t^{f-1}\right)\tilde{T}_2\left(x,t^{f-1}\right)dx\end{aligned} \tag{24}$$

where $\tilde{T}_2^*(\xi,x,t^f,t)$ is the interval fundamental solution for the mould sub-domain.

The numerical approximation of the equations (21) and (24) leads to the system of interval equations with the interval values of the elements of matrices **G** and **H**, this means

$$\begin{bmatrix} -\tilde{H}_{11}^{1} & -\tilde{H}_{12}^{1} & \tilde{G}_{12}^{1} & 0 \\ -\tilde{H}_{21}^{1} & -\tilde{H}_{22}^{1} & \tilde{G}_{22}^{1} & 0 \\ 0 & -\tilde{H}_{11}^{2} & \tilde{G}_{11}^{2} & -\tilde{H}_{12}^{2} \\ 0 & -\tilde{H}_{21}^{2} & \tilde{G}_{21}^{2} & -\tilde{H}_{22}^{2} \end{bmatrix} \begin{bmatrix} \tilde{T}_1(0, t^f) \\ \tilde{T}_1(L_1, t^f) \\ \tilde{q}(L_1, t^f) \\ \tilde{T}_2(L_2, t^f) \end{bmatrix} = \begin{bmatrix} -\tilde{G}_{11}^{1} \cdot q_L \\ -\tilde{G}_{21}^{1} \cdot q_L \\ -\tilde{G}_{12}^{2} \cdot q_R \\ -\tilde{G}_{22}^{2} \cdot q_R \end{bmatrix} + \begin{bmatrix} \tilde{P}_1(0, t^{f-1}) \\ \tilde{P}_1(L_1, t^{f-1}) \\ \tilde{P}_2(L_1, t^{f-1}) \\ \tilde{P}_2(L_2, t^{f-1}) \end{bmatrix} + \begin{bmatrix} \tilde{Z}_1(0, t^{f-1}) \\ \tilde{Z}_1(L_1, t^{f-1}) \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

where

$$\begin{aligned} &\tilde{G}_{11}^{e} = -\tilde{G}_{22}^{e} = -\frac{\sqrt{\Delta t}}{\sqrt{\tilde{\lambda}_e \tilde{c}_e \pi}} \\ &\tilde{G}_{12}^{e} = -\tilde{G}_{21}^{e} = \frac{\sqrt{\Delta t}}{\sqrt{\tilde{\lambda}_e \tilde{c}_e \pi}} \exp\left[-\frac{\left(L_e - L_{e-1}\right)^2}{4\tilde{a}_e \Delta t}\right] - \\ &\qquad \frac{L_e - L_{e-1}}{2\tilde{\lambda}_e} \operatorname{erfc}\left(\frac{L_e - L_{e-1}}{2\sqrt{\tilde{a}_e \Delta t}}\right), \qquad L_0 = 0 \end{aligned} \tag{26}$$

and

$$\begin{aligned} &\tilde{H}_{11}^{e} = \tilde{H}_{22}^{e} = -0.5 \\ &\tilde{H}_{12}^{e} = \tilde{H}_{21}^{e} = \frac{1}{2} \operatorname{erfc}\left(\frac{L_e - L_{e-1}}{2\sqrt{\tilde{a}_e \Delta t}}\right) \end{aligned} \tag{27}$$

while

$$\begin{aligned} &\tilde{P}_1(0, t^{f-1}) = \frac{1}{2\sqrt{\pi \tilde{a}_1 \Delta t}} \int_0^{L_1} \exp\left[-\frac{x^2}{4\tilde{a}_1 \Delta t}\right] \tilde{T}_1(x, t^{f-1}) \, \mathrm{d}x \\ &\tilde{P}_1(L_1, t^{f-1}) = \frac{1}{2\sqrt{\pi \tilde{a}_1 \Delta t}} \int_0^{L_1} \exp\left[-\frac{\left(x - L_1\right)^2}{4\tilde{a}_1 \Delta t}\right] \tilde{T}_1(x, t^{f-1}) \, \mathrm{d}x \\ &\tilde{P}_2(L_1, t^{f-1}) = \frac{1}{2\sqrt{\pi \tilde{a}_2 \Delta t}} \int_{L_1}^{L_2} \exp\left[-\frac{\left(x - L_1\right)^2}{4\tilde{a}_2 \Delta t}\right] \tilde{T}_2(x, t^{f-1}) \, \mathrm{d}x \\ &\tilde{P}_2(L_2, t^{f-1}) = \frac{1}{2\sqrt{\pi \tilde{a}_2 \Delta t}} \int_{L_1}^{L_2} \exp\left[-\frac{\left(x - L_2\right)^2}{4\tilde{a}_2 \Delta t}\right] \tilde{T}_2(x, t^{f-1}) \, \mathrm{d}x \end{aligned} \tag{28}$$

and

$$\begin{aligned} \tilde{Z}_1(0, t^{f-1}) = \frac{\sqrt{\Delta t}}{\sqrt{\tilde{\lambda}_1 \tilde{c}_1 \pi}} \int_0^{L_1} \tilde{Q}_1(x, t^{f-1}) \Bigg\{ &\exp\left[-\frac{x^2}{4\tilde{a}_1 \Delta t}\right] - \\ &\frac{|x|}{2\tilde{\lambda}_1} \operatorname{erfc}\left[\frac{x}{2\sqrt{\tilde{a}_1 \Delta t}}\right] \Bigg\} \mathrm{d}x \end{aligned} \tag{29}$$

$$\begin{aligned} \tilde{Z}_1(L_1, t^{f-1}) = \frac{\sqrt{\Delta t}}{\sqrt{\tilde{\lambda}_1 \tilde{c}_1 \pi}} \int_0^{L_1} \tilde{Q}_1(x, t^{f-1}) \Bigg\{ &\exp\left[-\frac{\left(x - L_1\right)^2}{4\tilde{a}_1 \Delta t}\right] - \\ &\frac{|x - L_1|}{2\tilde{\lambda}_1} \operatorname{erfc}\left[\frac{|x - L_1|}{2\sqrt{\tilde{a}_1 \Delta t}}\right] \Bigg\} \mathrm{d}x \end{aligned} \tag{30}$$

After determining the 'missing' boundary values for the casting and the mould sub-domains the interval temperatures $\tilde{T}_e^f$ at the internal points ξ^i can be calculated using the formulas
– for the casting sub-domain ($\xi \in (0, L_1)$)

$$\begin{aligned} &\tilde{T}_2\left(\xi, t^f\right) + \left[\frac{1}{\tilde{c}_2} \tilde{q}_2\left(x, t^f\right) \int_{t^{f-1}}^{t^f} \tilde{T}_2^*\left(\xi, x, t^f, t\right) dt\right]_{x=0}^{x=L_1} = \\ &\left[\frac{1}{\tilde{c}_2} \tilde{T}_2\left(x, t^f\right) \int_{t^{f-1}}^{t^f} \tilde{q}_2^*\left(\xi, x, t^f, t\right) dt\right]_{x=0}^{x=L_1} + \\ &\int_0^{L_1} \tilde{T}_2^*\left(\xi, x, t^f, t^{f-1}\right) \tilde{T}_2\left(x, t^{f-1}\right) dx \end{aligned} \tag{31}$$

– for the mould sub-domain ($\xi \in (L_1, L_2)$)

$$\begin{aligned} &\tilde{T}_2\left(\xi, t^f\right) = \frac{1}{2} \exp\left(-\frac{L_2 - \xi}{\sqrt{\tilde{a}_2 \Delta t}}\right) \tilde{T}_2\left(L_2, t^f\right) + \\ &\frac{1}{2} \exp\left(-\frac{\xi - L_1}{\sqrt{\tilde{a}_2 \Delta t}}\right) \tilde{T}_2\left(L_1, t^f\right) + \frac{\sqrt{\Delta t}}{2\sqrt{\tilde{\lambda}_2 \tilde{c}_2}} \exp\left(-\frac{L_2 - \xi}{\sqrt{\tilde{a}_2 \Delta t}}\right) \times \\ &q_R\left(L_2, t^f\right) + \frac{\sqrt{\Delta t}}{2\sqrt{\tilde{\lambda}_2 \tilde{c}_2}} \exp\left(-\frac{\xi - L_1}{\sqrt{\tilde{a}_1 \Delta t}}\right) \tilde{q}\left(L_1, t^f\right) + \tilde{P}_2\left(\xi, t^{f-1}\right) \end{aligned} \tag{32}$$

4. Interval Gauss elimination method

The interval Gauss elimination method [10, 11, 16] has been used to solve the interval system of equations (25). The obtained system of equations can be written in the following form

$$\tilde{\mathbf{A}} \cdot \tilde{\mathbf{y}} = \tilde{\mathbf{B}} \tag{33}$$

We use a decomposition to solve the linear system (33). The main matrix $\tilde{\mathbf{A}}$ must be written as a product of two matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{U}}$, where $\tilde{\mathbf{L}}$ is lower triangular and $\tilde{\mathbf{U}}$ is upper triangular, this means

$$\tilde{\mathbf{L}} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \tilde{L}_{21} & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{L}_{N1} & \tilde{L}_{N2} & \dots & 1 \end{bmatrix} \tag{34}$$

and

$$\tilde{\mathbf{U}} = \begin{bmatrix} \tilde{U}_{11} & \tilde{U}_{12} & \dots & \tilde{U}_{1N} \\ 0 & \tilde{U}_{22} & \dots & \tilde{U}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \tilde{U}_{NN} \end{bmatrix} \tag{35}$$

where $i, j = 1, 2, \dots, N$ (N is the dimension of the matrix).

The elements of the matrices $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{U}}$ are computed using the formulas according to the rules of the interval arithmetic, so

$$\begin{aligned} \tilde{U}_{ij} &= \tilde{A}_{ij} - \sum_{k=1}^{i-1} \tilde{L}_{ik} \tilde{U}_{kj} \quad \text{for} \quad j = i, i+1, \dots, N \\ \tilde{L}_{ji} &= \frac{1}{\tilde{U}_{ii}} \left(\tilde{A}_{ji} - \sum_{k=1}^{i-1} \tilde{L}_{jk} \tilde{U}_{ki} \right) \quad \text{for} \quad j = i+1, i+2, \dots, N \end{aligned} \tag{36}$$

and then the system of equations (33) takes a form

$$\tilde{\mathbf{A}} \cdot \tilde{\mathbf{y}} = (\tilde{\mathbf{L}} \cdot \tilde{\mathbf{U}}) \cdot \tilde{\mathbf{y}} = \tilde{\mathbf{B}} \tag{37}$$

As first, we solve for the vector **z** such that

$$\tilde{\mathbf{L}} \cdot \tilde{\mathbf{z}} = \tilde{\mathbf{B}} \tag{38}$$

and then we solve for the vector **y** the system as follows

$$\tilde{\mathbf{U}} \cdot \tilde{\mathbf{y}} = \tilde{\mathbf{z}} \tag{39}$$

5. Numerical examples

In this paper two examples of 1D heat transient transfer in a casting-mould system of dimensions L_1 = 0.02 [m] (casting) and L_2 = 0.02 [m] (mould) are presented. On the both sides the boundary condition of the 2nd type $q_L = q_R = 0$ [W/m^2] have been assumed. The casting sub-domain and the mould sub-domain have been divided into 20 constant internal cells, respectively.

The following input data have been introduced: λ_1 = 180 [W/(m·K)], c_1 = 3 [MJ/m^{3}K], λ_2 = 2.6 [W/(m·K)], c_2 = 1.75 [MJ/m^{3}K], pouring temperature T_{01} = 670 [°C], initial mould temperature T_{02} = 30 [°C], solidification point T_{cr} = 660 [°C], volumetric latent heat Q_{cr} = 975 [MJ/m^3], time step Δt = 0.02 [s].

In the first example the interval source function with the interval values of the growth coefficient $\tilde{\mu} = \langle 2.95 \cdot 10^{-6}, 3.05 \cdot 10^{-6} \rangle$ [m/s·K^2] and the grain density $\tilde{N} = \langle 9.8 \cdot 10^8, 10.2 \cdot 10^8 \rangle$ [1/m^3] is assumed.

Figure 2 presents the courses of the source function at nodes 10 ($x = 0.0095$ [m]) and 15 ($x = 0.0145$ [m]) from the casting sub-domain, where SourceL and SourceR denote the first and the second endpoints of the source interval.

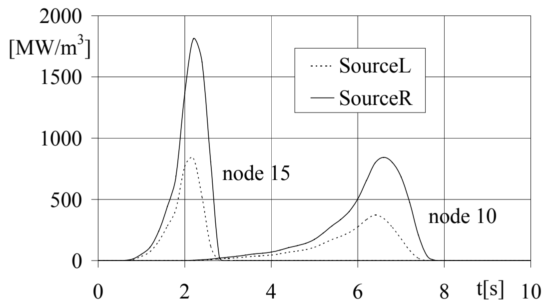

Fig. 2. The courses of the source function

Figure 3 illustrate the temperature distribution in the casting-mould domain obtained for the time 5[s] (TemL and TemR denote the first and the second endpoints of the temperature interval).

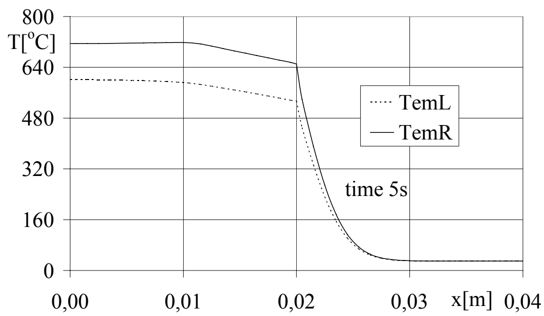

Fig. 3. The temperature distribution

In the second example the volumetric specific heat of the mould sub-domain is assumed to be an interval value $\tilde{c}_2 = \langle c_2 - 0.01 \cdot c_2, c_2 + 0.01 \cdot c_2 \rangle$. Figure 4 and 5 illustrate the temperature distribution in the casting-mould domain obtained for the time 5[s] and 10[s].

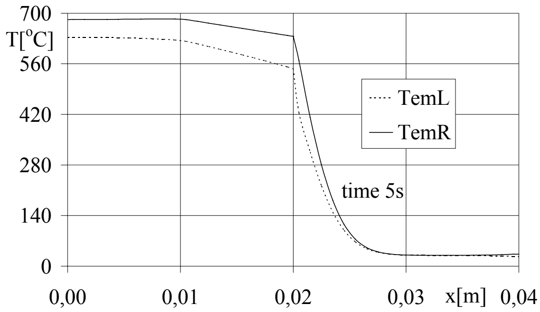

Fig. 4. The temperature distribution for the time 5[s]

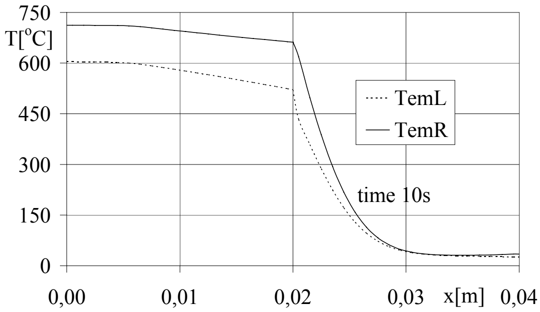

Fig. 5. The temperature distribution for the time 10[s]

6. Conclusions

In this paper the crystallization of pure metal in the sand mould is analysed. The growth coefficient, grain density and all remaining thermophysical parameters have been assumed as interval values. The problem discussed has been solved using the 1st scheme of the interval boundary element method.

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