ARCHIVES of FOUNDRY ENGINEERING

Published quarterly as the organ of the Foundry Commission of the Polish Academy of Sciences

ISSN (1897-3310)
Volume 11
Issue 1/2011

31 – 34

6/1

Description of the particle distribution in the space of composite suspension casting by statistical methods

J. Grabian, K. Gawdzińska*, W. Przetakiewicz, M. Pijanowski
Institute of Basic Technical Sciences, Maritime University of Szczecin,
ul. Podgórna 51/53, 70-205 Szczecin, Polska
* Corresponding author. E–mail address: k.gawdzinska@am.szczecin.pl

Received 02.03.2011; accepted in revised form 16.03.2011

Abstract

This article presents a description of the reinforcement phase distribution in the space of composite suspension casting. The statistical methods used include Spearman's rank correlation coefficient combined with the significance test, Chi-square test of independence and the test of contingency. The reinforcement phase consisted of SiC particles (15% by weight, and the matrix was AlSi11 alloy). Composites were made by mechanical stir casting method.

Keywords: Composite casting; Homogeneity; Statistics

1. Introduction

The structure of composite materials can be described as a composition of two structural components, known as the matrix and reinforcement [1–5]. Composite materials produced by casting methods often have non-homogeneous distribution and content of reinforcement in the casting volume, with varied shape and size of reinforcement particles [1, 3]. It clearly follows from the knowledge of materials engineering that this type of structural differentiation substantially influences material properties. Therefore, a need arises to consistently describe the characteristics of composite structure – in this case the distribution of reinforcement in the casting volume. Such description can be done, comparing selected areas of the examined material, by statistical methods [6-7]. This work is limited to cast metallic composites that due to their manufacturing technology are liable to have non-homogeneous distribution of the reinforcement phase.

2. Research

The authors describe the distribution of the reinforcement phase particles in the space of composite suspension casting (AlSi11/SiC), made by the mechanical stir method [3]. The composites were cast in a metal mould of a cylindrical shape, 48 mm in diameter and 100 mm high. The mould was gravitationally filled up.

Three areas, or samples, were tested (Figs 3-5, numbered 1, 2, 3) taken from the material under examination. The places of sampling are shown in Fig 1. They are compared with a generated model structure (Fig 2), marked by the letter M. The visualization of relationships between the variables, describing the reinforcement phase in the tested areas, are depicted using the computer software STATISTICA PL [8].

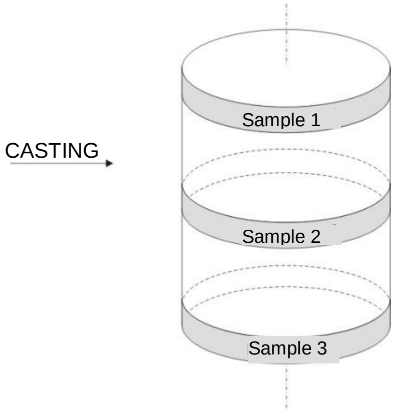

Fig. 1. Areas of sampling for the analysis of the reinforcement structure homogeneity in the casting space

In order to assess the non-homogeneity of particle positions, two properties defining the particle distribution in the sample of were considered:

- horizontal position, i.e. the variable X as the abscissa in the Cartesian system, and
- vertical position, the variable Y, the ordinate in the Cartesian system.

The level of variables X and Y independence was assumed as the measure of non-homogeneity, as in a homogeneous sample the position of particles should have a uniform random distribution, therefore the variables X and Y have to be independent.

The diagrams below (Figs. 2–5) present the distribution of particles in respective samples.

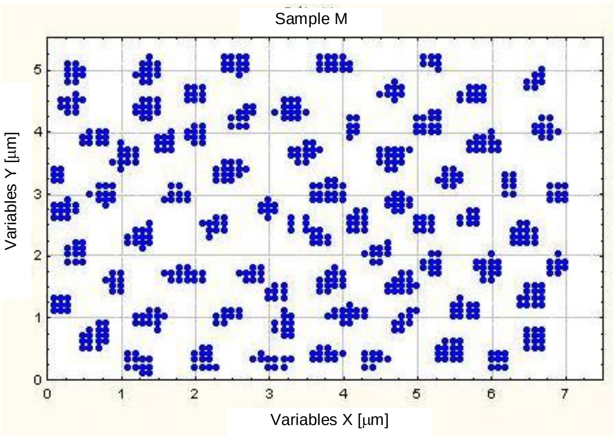

Fig. 2. Visualization of particles distribution in the model sample, made by means of [8]

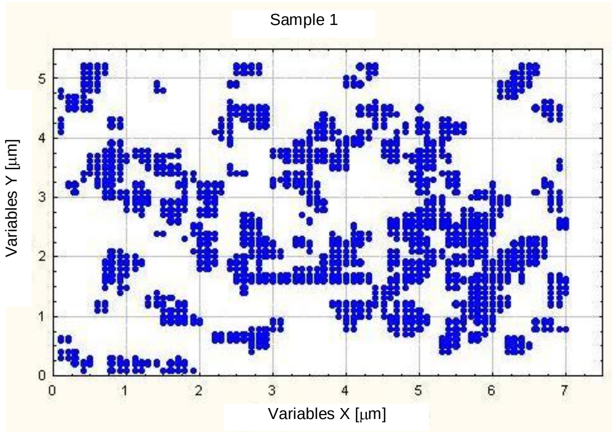

Fig. 3. Visualization of particles distribution in the sample 1, made by means of [8]

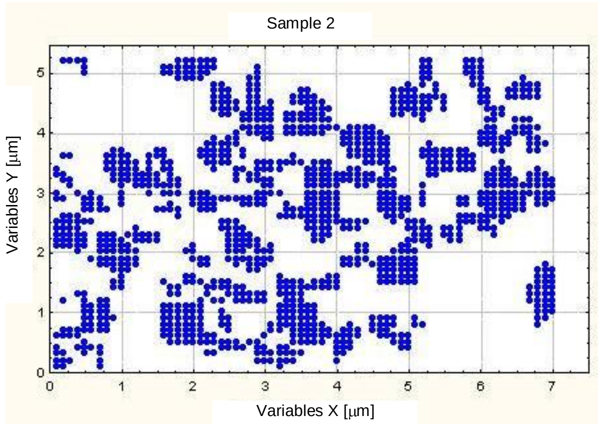

Fig. 4. Visualization of particles distribution in the sample 2, made by means of [8]

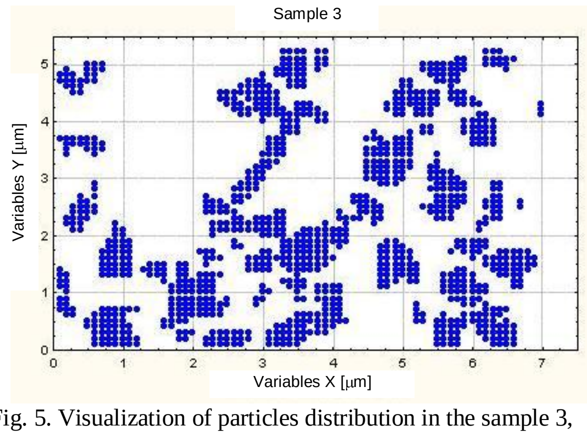

Fig. 5. Visualization of particles distribution in the sample 3, made by means of [8]

Three statistical methods will be used for the assessment of position non-homogeneity [9-11], namely:

- Spearman's rank correlation coefficient together with the significance test,
- Chi-square test of independence,

– coefficient of contingency.

Spearman's rank correlation coefficient denoted as r_S is a measure of relationship between qualitative random variables determined for the ranks of variable values, not for the values themselves of these variables. The coefficient is calculated as follows:

– we order the values of the properties for n statistical units;
– we assign ranks to these values, i.e. subsequent natural numbers;
– we determine d_i, the difference between two respective ranks of both properties for i-th statistical unit.

Spearman's rank correlation coefficient is derived from this formula [9-13]:

$$r_S = 1 - \frac{6 \cdot \sum_{i=1}^{n} d_i^2}{n \cdot (n^2 - 1)}$$

The coefficient assumes values comprised on the interval [–1,1]. If the correlation coefficient is 0, then there is no relationship between the properties; if the coefficient is found on the [–0.3 – 0.3] interval (except for 0) there is a slight (weak) correlation; for intervals –0.3 to –0.5 and 0.3 to 0.5 the correlation is visible (quite strong); the intervals –1 to –0.5 and 0.5 to 1 mean a very strong correlation.

Additionally, a significance test was performed, which verifies the null hypothesis:

$H_0 : r_S = 0$

against an alternative hypothesis:

$H_1 : r_S \neq 0$.

The test statistic t is then calculated and the critical level of test significance p is determined. With the fixed test significance level α, if $p \le \alpha$ we reject the null hypothesis and accept the alternative hypothesis (statistically, the correlation coefficient is significantly different from zero). If $p > \alpha$ there are no grounds to reject the null hypothesis (statistically, the correlation coefficient does not differ significantly from zero). Table 1 contains the calculation results.

Table 1.
The values of Spearman's rank correlation coefficient, test statistic t and critical significance level p for the examined samples, calculated by means of [8]

	Sample M	Sample 1	Sample 2	Sample 3
Spearman's rank correlation coefficient	-0.074	-0.096	0.216	0.130
Test statistic t	-2.07	-3.17	7.68	4.31
Critical level of significance p	0.0386	0.00158	0.0000	0.0000

When the test significance level is set at $\alpha = 0.01$, there are no grounds to reject the null hypothesis for the model sample, which means that the correlation coefficient does not differ significantly from zero in terms of statistics. However, the remaining Spearman's rank correlation coefficients statistically differ from zero significantly, therefore we reject the null hypothesis in favour of the alternative hypothesis. The most non-homogeneous samples are the samples 2 and 3, since their correlation coefficients are above 0.1.

As the next step, the Chi-square test of independence (χ^2) was used for the assessment of the particle distribution non-homogeneity in a sample. The test allows to verify the null hypothesis:

H_0: X and Y are independent,
against the alternative hypothesis:
H_1: X and Y are dependent.

The test statistic in this test is χ^2 (Chi-square) statistic, for which the critical significance test level p is determined. With the fixed test significance level α, if $p \le \alpha$ we reject the null hypothesis and accept the alternative hypothesis (variables X and Y are dependent), and if $p > \alpha$ there are no grounds to reject the null hypothesis (variables X and Y are independent).

The results of calculated test statistics χ^2 and critical levels of test significance for each sample are given in Table 2:

Table 2.
Test statistics χ^2 and critical levels of test significance for the samples, calculated by means of [8]

	Sample M	Sample 1	Sample 2	Sample 3
Test statistic χ^2	46.218	190.695	196.700	237.809
Critical significance level p	0.0386	0.0000	0.0000	0.0000

With the fixed level of test significance α=0.01 there are no grounds to reject the null hypothesis [12-15] for the model sample, i.e. the variables X and Y describing the position of particles in the model sample are independent. For the other samples the null hypothesis is rejected in favour of the alternative hypothesis, which means that the variables X and Y describing the position of particles in the samples are dependent.

The coefficient of contingency, based on the values of Chi-square statistic, is a measure of correlation between the two categorized qualitative variables as proposed by Pearson, the creator of the Chi-square test. The advantage of this measure, compared to the normal Chi-square value, is that its values belong to the [0,1] interval, where 0 means the variables are independent, and it can be interpreted as the coefficient of rank correlation. The values of contingency coefficient for each sample are presented in Table 3.

Table 3.
Values of the coefficient of contingency for the samples, calculated by means of [8]

	Sample M	Sample 1	Sample 2	Sample 3
Coefficient of contingency	0.2374	0.3869	0.3740	0.4243

The values given in Table 3 allow to state that the sample 3 is the most non-homogeneous in terms of particle positioning, while the sample 2 is the least non-homogeneous.

3. Summary

The above attempt to describe the distribution of the reinforcement phase in the space of composite suspension casting with the application of statistical methods can be employed as a tool for the determination of this characteristic feature, which, in turn, will contribute to quality improvement of manufactered composite materials.

References

[1] S. Wong, E. Neussl, D. Fettweis, O.R. Sahm, H.M.L. Flower, High strength Al-Zn-Mg matrix-alloy for continuous fibre reinforcement, [in:] Metal matrix composites and metallic foams, 2000, Euromat'2000; vol. 5, Wiley-Vch Verlag Gmbh 2000, 119–127.

[2] J. Sobczak, Kompozyty metalowe, Instytut Odlewnictwa i Instytut Transportu Samochodowego, Kraków – Warszawa 2001.

[3] J. Śleziona, Podstawy technologii kompozytów, Wyd. Politechniki Śląskiej, Gliwice 1998.

[4] W. Orłowicz, A. Trytek, Shaping of microstructure and service properties of cast iron casting by surface refinement with electric arc plasma, Archives of Foundry Engineering, Vol. 7, No. 23(2007), 120.

[5] K. Gawdzińska, M. Maliński, Study of Reinforcement Elements Distribution Exemplified by Composite with AlSi11 Matrix and Carbon Reinforcement, Metallurgy, vol. 44 (2005), 1, str. 45–48.

[6] M. Sobczyk, Statystyka, Wydawnictwo Naukowe PWN, Warszawa 2002.

[7] K. Gawdzińska, J. Szala, M. Maliński, Computer programs to model three dimensional structure of the MMC, Archives of Foundry Engineering, Volume 9, issue 4/2009, 2009, s. 53–59.

[8] Program komputerowy STATISTICA PL.

[9] J. Greń, Statystyka matematyczna. Modele i zadania, PWN, Warszawa 1987.

[10] S. Brandt, L. Szymanowski: Analiza danych : metody statystyczne i obliczeniowe. Warszawa: Wydawnictwo Naukowe PWN, 1998

[11] K. Gawdzińska, J. Szala, M. Maliński, Inhomogeneity of reinforcement arrangement in metallic composites, Inżynieria Materiałowa Nr 4/2008, 211–214.

[12] C. Domański, Testy statystyczne, PWE, Warszawa 1999.

[13] M. Maliński Kryteria doboru procedur statystycznych – ilustracja na przykładach zagadnień stereologii i analizy obrazu, Wiadomości stereologiczne, październik, 2003, s. 18–20.

[14] M. Maliński, Statystyka matematyczna wspomagana komputerowo, Wydawnictwo Politechniki Śląskiej 2010.

[15] M. Maliński, Weryfikacja hipotez statystycznych wspomagana komputerowo, Wydawnictwo Politechniki Śląskiej. Gliwice 2004.