3D model of impulse compaction of moulding sands model

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Abstract

A mathematical 3D model is presented, fully describing the impulse compaction process of moulding sands. The model is based on the models of the impulse head dynamics as well as of the deformation and compaction processes of moulding sand. Deformation and compaction processes were modelled on the grounds of the viscoelastic rheological model of moulding sand. It was found that knowing the and coefficients characterising viscous and elastic properties of moulding sand makes the necessary and sufficient condition for simulation testing of the developed model. The coefficients can be determined by ultrasonic testing of moulding sand. The simulation and experimental research of the impulse compaction process proved that the developed model describes the impulse compaction process very well.

Keywords: Moulding sand, Impulse compaction process, Rheological model, 3D mathematical model

1. Introduction

Designing and optimising the impulse compaction process of moulding sands requires knowing its mathematical model and the results of simulation testing of that model. Unfortunately, due to the complexity of the phenomena which occur during impulse compaction of moulding sands, a mathematical model fully describing this process has not been developed yet, although many researchers in the world entered upon such modelling, among others Boenisch et al. [1], Orlov et al. [2], Bast et al. [3], and Smyksy et al. [4].

No one of the so far developed model including a complex description of the impulse compaction process of moulding sands and, as a result of the assumed excessive simplifications, simulation testing of those models give no quantitative information on the impulse compaction process, even though in simplified way. So, the so far published models and their testing have not found a practical application in design and optimisation of the impulse compaction process of moulding sands. In the Basic Automation Laboratory of the Institute of Production Engineering and Automation, Wrocław University of Technology, research works on mathematical modelling as well as simulation and experimental testing of the impulse compaction process of moulding sands have been carried out for many years. As a result, a mathematical 3D model has been developed, fully describing the impulse compaction process of moulding sands [5]. The ground for the model was made by:

- mathematical description of dynamics of the impulse moulding head,
- mathematical model of the impulse compaction process of moulding sands formulated using rheological description of mechanical properties of moulding sands.
2. 3D mathematical model of the impulse compaction process

As mentioned above, formulating a mathematical model of the impulse compaction process requires knowing the models of:
- the impulse head,
- the impulse compaction process of moulding sand.

For impulse compaction of moulding sands, the head developed in the Basic Automation Laboratory can be used. Layout of the impulse compaction head with a self-acting impulse valve is shown in Fig. 1.

![Fig. 1. Layout of the impulse compaction head controlled by a self-acting impulse valve: 1 – distributor valve; 2 – accumulator tank; 3 – self-acting impulse valve](image)

To describe dynamics of the impulse head with a self-acting impulse valve, a model created by Gerc et al. [6,7] and modified by the authors was applied. The model was developed assuming the following simplifying assumptions: air is a perfect gas, thermodynamical processes are of quasistatic nature, there is no heat exchange between the gas in the drive chambers and the environment, frictional resistance in sealings are negligibly low. Considering the above simplifying assumptions, dynamics of the impulse head can be described by the following system of differential equations:

\[
m_t \cdot \frac{d^2 y}{dt^2} = F_1 \cdot (p_a - p_1) - c \cdot (y + y_0) - m_t \cdot g \quad (1)
\]

\[
\frac{dp_a}{dt} = -\frac{\kappa \cdot G_s \cdot R \cdot T_i}{V_0} \quad (2)
\]

\[
\frac{dp_1}{dt} = \frac{\kappa}{h + x_1} \left( \frac{G_s \cdot R \cdot T_i}{F_1} - p_1 \cdot \frac{dx_1}{dt} \right) \quad (3)
\]

\[
\frac{dp_2}{dt} = \frac{\kappa \cdot sz - y}{p_1 \cdot \frac{dy}{dt}} \left( \frac{G_s \cdot R \cdot T_i}{F_2} \right) \quad (4)
\]

where:
- \( m_t \) – mass of the impulse valve piston,
- \( y \) – co-ordinate of the impulse valve piston,
- \( F_1 \) – cross-section area of the moulding box,
- \( F_2 \) – cross-section area of the impulse valve piston,
- \( p_0, p_1, p_2 \) – absolute pressure in the accumulator chamber, in the working space above the moulding sand and in the impulse valve return chamber, respectively,
- \( c \) – spring constant,
- \( y_0 \) – preliminary (assembly) deflection,
- \( g \) – acceleration of gravity,
- \( \kappa \) – adiabate exponent,
- \( G_i \) – air flow rate from \( i \)th chamber,
- \( R \) – gas constant of the air,
- \( T_i \) – air temperature in \( i \)th chamber,
- \( V_0 \) – volume of the accumulator tank,
- \( h \) – distance between the head and the upper layer of the mass column,
- \( x_1 \) – co-ordinate of the upper layer of the mass column subject to compaction,
- \( sz \) – stroke of the impulse valve

The relationship describing the air flow rate from the \( i \)th chamber is:

\[
G_i = K \cdot \alpha_i \cdot f_i \cdot p_{a,1} \cdot \frac{1}{\sqrt{R \cdot T_i}} \cdot \varphi(\varepsilon_i) \quad (5)
\]

where:
- \( \alpha_i \) – air flow rate coefficient,
- \( f_i \) – flow area of the choke,
- \( \varepsilon_i = \frac{p_{a,1}}{p_i} \cdot \kappa = \frac{2 \cdot \kappa}{\kappa - 1} \)
- \( p_i \) – pressure in the working chamber,
- \( p_{a,1} \) – atmospheric pressure or pressure in the working chamber,
- \( \varphi(\varepsilon_i) = \begin{cases} 
0.2588 & \text{for } 0 < \varepsilon_i \leq 0.53 \\
\sqrt{\frac{\varepsilon_i \cdot \varepsilon_i}{\varepsilon_i - 0.53}} & \text{for } 0.53 < \varepsilon_i \leq 1 
\end{cases} \quad (6)
\]

The equations (1) to (6) have the following meanings:
- Equation (1) describes the impulse valve piston motion.
- Equations (2) to (4) are models of gas transformation in the accumulator chamber, in the working space above moulding sand and in the impulse valve return chamber, respectively.
- Equations (5) and (6) describe the relationships related to air flow from the \( i \)th chamber.

The deformation and compaction process of moulding sand can be formulated on the basis of a known rheological model of the sand. It was proved in the papers [8-11] that rheological properties of moulding sand can be described by a viscoelastic rheological model. The suggested model, schematically shown in Fig. 2, permits description of the process of impulse compaction of moulding sand in any layer of the mould.
Fig. 2. Diagram of the three-dimensional model of moulding sand

Fig. 3. Diagram of the process of impulse compaction of moulding sand: 1 – distributor valve; 2 – accumulator tank; 3 – self acting impulse valve; 4 – moulding sand

Considering the model of the impulse head dynamics and the rheological model of moulding sand, the process of impulse compaction of moulding sand (Fig. 3) can be described by the following system of differential equations:

\[ g \cdot m \cdot \ddot{x}_i + k_i(x_i) \cdot (x_i - x_{i-1}) + k_i(x_i) \cdot (x_i - x_{i+1}) = p_1 \cdot F_i + m_i \cdot g \]  \hspace{1cm} (7)

\[ m \cdot \ddot{x} + k(x_i) \cdot (x_i - x_{i-1}) + k(x_i) \cdot (x_i - x_{i+1}) + k(x_i) \cdot (x_i - x_{i-1}) + k(x_i) \cdot (x_i - x_{i+1}) = m_i \cdot g \]  \hspace{1cm} (8)

\[ m \cdot \ddot{x} + k(x_i) \cdot (x_i - x_{i-1}) + k(x_i) \cdot (x_i - x_{i+1}) + k(x_i) \cdot (x_i - x_{i-1}) + k(x_i) \cdot (x_i - x_{i+1}) = m_i \cdot g \]  \hspace{1cm} (9)

\[ m \cdot \ddot{x} + k(x_i) \cdot (x_i - x_{i-1}) + k(x_i) \cdot (x_i - x_{i+1}) + k(x_i) \cdot (x_i - x_{i-1}) + k(x_i) \cdot (x_i - x_{i+1}) = m_i \cdot g \]  \hspace{1cm} (10)

\[ p_{ci} (\delta) = \frac{k_i(x_i \cdot (x_i(t) - x_{i-1}(t)) + k_i(x_i \cdot (x_i(t) - x_{i+1}(t))) \cdot p_i} \]  \hspace{1cm} (11)

where:
- \( m_i \) – mass of the \( i^{th} \) layer of moulding sand,
- \( x_i \) – co-ordinate of the \( i^{th} \) layer,
- \( p_1 \) – pressure in the working space above moulding sand,
- \( F_i \) – cross-section area of the moulding box,
- \( p_{ci} \) – total pressure in the \( i^{th} \) layer of moulding sand,
- \( p_{ui} \) – pressure in \( i^{th} \) layer resulting from compaction of moulding sand.

The equations (7) to (11) have the following meanings:

- Equations (7) to (10) describe the deformation process in moulding sand, respectively in 1st, 2nd, ... nth layer,
- Equation (11) describes change of pressures in the \( i^{th} \) layer of moulding sand as a function of time.

3. Testing results of the 3D model of the impulse compaction process

Application of the presented mathematical model for simulation tests in the Matlab-Simulink environment requires only knowing the parameters characterising viscous and elastic properties of moulding sand, i.e. the coefficients \( k_i = f(x_i) \) and \( k_i = f(x) \). These coefficients can be determined by ultrasonic testing, as described in [11].

Figure 4 shows the results of simulation testing, which represent changes in dynamically squeezed moulding sand on three selected heights of the sand column. In these tests, initial height of the moulding sand was \( H_p = 350 \) mm and supply pressure was \( p_0 = 0.4 \) MPa and \( p_0 = 0.5 \) MPa.

Figure 5 shows changes of total pressure \( p_c = f(t) \), determined on the basis of simulation and experimental examination of the impulse compaction process at a selected height of the moulding sand column.

![Figure 4](image)

![Figure 5](image)
insignificant differences of pressures in moulding sand in steady states.

4. Conclusion

A 3D mathematical model was presented, fully describing the process of impulse compaction of moulding sands, formulated on the grounds of a description of the impulse head dynamics and a rheological model of deformation and compaction of moulding sand.

The process of moulding sand deformation and compaction was described on the ground of a rheological model of the sand, being a series combination of several elementary viscoelastic models. This permitted modelling the deformation and compaction process in any volume of the sand column.

The developed three–dimensional model of impulse compaction of moulding sands for the first time made it possible to obtain simulation results of moulding sand impulse compaction in any volume of the compacted moulds.

Analysis of the presented simulation and experimental results allows the statement that the developed 3D model describes the impulse compaction process of moulding sands very well not only in qualitative way, but first of all in quantitative way.

References