SENSITIVITY ANALYSIS OF CONTINUOUS CASTING WITH REGARD TO TIME-DEPENDENT POURING TEMPERATURE

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SUMMARY

In the paper the sensitivity analysis of temperature field in domain of cast strand with respect to the heat transfer coefficient on the outer surface is presented. The pouring temperature is assumed to be time-dependent – it results from the real technological conditions. The problem is treated as a transient and non-linear one. It causes that the equations determining the sensitivity field become more complex as in the case of typical sensitivity models. On the basis of solution obtained one can analyse the changes of temperature field resulting from the changes of cooling conditions. On the stage of numerical simulations the control volume method has been used.

Key words: continuous casting, numerical modelling, sensitivity analysis

1. GOVERNING EQUATIONS

The thermal processes proceeding in domain of rectangular, vertical continuous casting are described by the following energy equation

\[ C(T) \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) \]

where \( T = T(x, y, z, t) \), \( C \) is the volumetric substitute thermal capacity \([1,2]\), \( \lambda \) is the thermal conductivity, \( u \) is the pulling rate.
The boundary conditions on the outer surface of the cast strand are of the form of Neumann (boundary heat flux) or the Robin (continuity of boundary heat flux). In this paper the Robin condition are taking into account [3,4]. So

\[-\lambda \frac{\partial T}{\partial n} = \alpha (T - T_w)\]  

(2)

where \(\alpha\) is the heat transfer coefficient, \(T_w\) is the cooling water temperature. On the upper surface of the cast strand the Dirichlet condition is given:

\[T = T_p(t)\]  

(3)

where \(T_p\) is the time-dependent pouring temperature.

On the conventionally assumed lower surface of the system the no-flux condition \((\partial T / \partial n = 0)\) can be accepted. The initial condition reduces to the assumption that the certain layer of molten metal directly beyond the starting bar has the pouring temperature - Fig.1.

![Fig.1. Initial condition](image)

We consider the 2D problem corresponding to section parallel to the shorter side of cast strand (a plane of geometrical symmetry) and then the equation (1) takes a form

\[C(T) \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right)\]  

(4)
2. SENSITIVITY ANALYSIS

We will consider the thermal processes proceeding in the primary cooling zone where the influence of time-dependent pouring temperature is especially visible. Thermal interactions between the casting and crystallizer are determined by the Robin condition in which the substitute heat transfer coefficient resulting from the measurements [3,4] appears. Its value is from the scope 1200-1700 [W/m²].

The sensitivity analysis with respect to \( \alpha \) requires the differentiation of equation (4) over this parameter:

\[
\frac{\partial C(T)}{\partial \alpha} \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] + C(T) \frac{\partial}{\partial \alpha} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right) = 0
\]

\[
\frac{\partial}{\partial x} \left[ \frac{\partial \lambda(T)}{\partial \alpha} \frac{\partial T}{\partial x} + \lambda(T) \frac{\partial}{\partial \alpha} \left( \frac{\partial T}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[ \frac{\partial \lambda(T)}{\partial \alpha} \frac{\partial T}{\partial z} + \lambda(T) \frac{\partial}{\partial \alpha} \left( \frac{\partial T}{\partial z} \right) \right]
\]

In this place the Schwarz theorem and the method of complex derivative calculation should be used, additionally we denote \( \frac{\partial T}{\partial \alpha} = U \) and then

\[
\frac{dC(T)}{dT} U \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] + C(T) \left( \frac{\partial U}{\partial t} + u \frac{\partial U}{\partial z} \right) = 0
\]

\[
\frac{\partial}{\partial x} \left[ \frac{d \lambda(T)}{dT} U \frac{\partial T}{\partial x} + \lambda(T) \frac{\partial U}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{d \lambda(T)}{dT} U \frac{\partial T}{\partial z} + \lambda(T) \frac{\partial U}{\partial z} \right]
\]

The last equation can be written in the form

\[
C(T) \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) -
\]

\[
\frac{dC(T)}{dT} U \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] + \frac{\partial}{\partial x} \left[ \frac{d \lambda(T)}{dT} U \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{d \lambda(T)}{dT} U \frac{\partial T}{\partial z} \right]
\]

or

\[
C(T) \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + Q_U
\]
where

\[ Q_U = -\frac{dC(T)}{dT} U \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] + \frac{\partial}{\partial x} \left[ \frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial z} \left[ \frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial z} \right] \] (9)

The function \( Q_U \) corresponds to the source function in the typical Fourier-Kirchhoff equation. If we assume the constant value of thermal conductivity \( \lambda \) then the source function becomes simpler:

\[ Q_U = -\frac{dC(T)}{dT} U \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right] \] (10)

Differentiation of the Robin condition gives

\[ -\frac{\partial \lambda(T)}{\partial \alpha} \frac{\partial T}{\partial n} - \lambda(T) \frac{\partial}{\partial \alpha} \left( \frac{\partial T}{\partial n} \right) = T - T_w + \alpha \frac{\partial T}{\partial \alpha} \] (11)

After the mathematical manipulations we obtain

\[ -\frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial n} - \lambda(T) \frac{\partial U}{\partial n} = T - T_w + \alpha U \] (12)

or

\[ -\lambda(T) \frac{\partial U}{\partial n} = \frac{d\lambda(T)}{dT} U \frac{\partial T}{\partial n} + T - T_w + \alpha U \] (13)

For \( \lambda = \text{const} \) we have

\[ -\lambda \frac{\partial U}{\partial n} = \alpha U + T - T_w = \alpha (U - U_w) \] (14)

where

\[ U_w = \frac{T_w - T}{\alpha} \] (15)

The mathematical model of sensitivity with respect to \( \alpha \) is supplemented by the transformed initial condition and finally we obtain the following additional boundary-initial problem (for \( \lambda = \text{const} \)
\[
\begin{cases}
(x, z) \in \Omega : & C(T\left[ \frac{\partial U}{\partial t} + u \frac{\partial U}{\partial x} \right] = \frac{\partial}{\partial x}\left( \lambda \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial z}\left( \lambda \frac{\partial U}{\partial z} \right) + Q_v \\
(x, z) \in \Gamma : & -\lambda \frac{\partial U}{\partial n} = \alpha(U - U_0) \\
t = 0 : & U = 0
\end{cases}
\]  

(16)

where the source function results from (10).

The knowledge of the sensitivity field allows to transform the basic solution (obtained for \( \alpha = \alpha_0 \)) on the solution corresponding to \( \alpha = \alpha_0 \pm \Delta \alpha \). It results from the Taylor formula, namely

\[
T(x, z, t, \alpha) = T(x, z, t, \alpha_0) \pm U(x, z, t, \alpha_0) \Delta \alpha
\]

(17)

In order to solve the basic problem and also the sensitivity model the control volume method has been used.

3. THE CONTROL VOLUME METHOD

In Fig.2 the 3D control volume is shown. The energy balance can be written in the form

\[
C_0 \left( T_0^{f+1} - T_0^f \right) V_0 = \sum_{e=1}^{s} Q_{0e}^f + q_v^f \Delta V_0 \Delta t
\]

(18)

where \( Q_{0e} \) is the heat conducted from the control volume \( \Delta V_0 \) to the adjoining volumes \( \Delta V_e \), \( e = 1, \ldots, 12s \) where \( s \) is the problem dimension, \( q_v \) is the heat emitted inside the volume \( \Delta V_0 \) during the time \( \Delta t = t^{f+1} - t^f \), \( T_0 \) is the temperature at the centre of control volume \( \Delta V_0 \).

The quantities of \( Q_{0e} \) are equal to (c.f. Fig.2)

\[
Q_{0e}^f = \frac{T_e^f - T_0^f}{R_e^f} \Delta A_e \Delta t, \quad e = 1, 2
\]

(19)

and

\[
Q_{03}^f = \frac{T_3^f - T_0^f}{R_3^f} \Delta A_3 \Delta t + C_0^f u T_0^{f+1} \Delta A_3 \Delta t
\]

(20)
Let us assume that \( \Delta z = u \Delta t \). Then after mathematical manipulations one obtain the following equation

\[
T_4^{f+1} = \frac{4}{\sum_{e=0}^{W_e}} T_e^f + \frac{q_{v_e} \Delta t}{C_0}
\]

where

\[
W_e = \frac{\Delta t \Delta A_e}{\sum_{e=0}^{4} C_0 R_e \Delta V_0}, \quad e > 0, \quad W_0 = 1 - \sum_{e=1}^{4} W_e
\]
In order to assure the stability of such scheme we must to determine the time interval fulfilling the condition $W_0 > 0$ for the all control volumes.

4. EXAMPLE OF COMPUTATIONS

The cast stand of dimensions $0.6 \times 0.2$ [m] made from carbon steel has been considered. The initial pouring temperature $T_p(0) = 1550$ [$^\circ$C] decreases with rate $2$[K/min], pulling rate: $u = 0.017$ [m/s]. The thermophysical parameters of casting material have been taken from [5], at the same time the constant value of thermal conductivity $\lambda = 35$ [W/mK] has been assumed. The basic heat transfer coefficient for the primary cooling zone equals $\alpha = 1500$ [W/m$^2$K], cooling water temperature $T_w = 40$ [$^\circ$C]. On the stage of sensitivity analysis application the change of $\alpha$ was assumed to be $\Delta \alpha = 200$ [W/m$^2$K].

The basic solution ($\alpha = 1500$ [W/m$^2$K]) and the solution for $\alpha = 1700$ [W/m$^2$K] have been found using the same computer program. Next the sensitivity field has been found (the problem (16)) and the solution for $\alpha = 1500$ [W/m$^2$K] has been rebuilt on the solution corresponding to $\alpha = 1700$ [W/m$^2$K] – see: equation (17). The comparison of the results obtained is presented below.

Table 1. The direct (left) and the indirect (right) solutions (the temperatures after 400s at the same points).

<table>
<thead>
<tr>
<th></th>
<th>Direct (po lewej)</th>
<th>Indirect (po prawej)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1546</td>
<td>1540 1529 1510 1479 1250</td>
<td>1545 1539 1528 1510 1478 1248</td>
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<tr>
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<td>1539 1527 1508 1474 1243</td>
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<td>1543</td>
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<tr>
<td>1542</td>
<td>1535 1522 1503 1377 1214</td>
<td>1541 1534 1521 1502 1375 1212</td>
</tr>
<tr>
<td>1541</td>
<td>1533 1520 1500 1375 1194</td>
<td>1540 1532 1519 1498 1372 1192</td>
</tr>
</tbody>
</table>

Summing up is seems that the sensitivity analysis gives the new possibilities in the numerical computations of the typical problems from the scope of the thermal theory of foundry processes.

REFERENCES

ANALIZA WRAŻLIWOŚCI CIĄGŁEGO ODlewania NA ZMIANĘ TEMPERATURY ZALEWANIA W FUNKCJI CZASU

STRESZCZENIE

W pracy przedstawiono analizę wrażliwości nieustalonego pola temperatury w objętości wlewka ciągłego na zmiany współczynnika wymiany ciepła na jego powierzchni. Załozono zmienną, zależną od czasu temperaturę zalewania. Równanie opisujące procesy cieplne w obszarze wlewka jest nieliniowe, co powoduje, że równania wrażliwości różnią się od klasycznych. Otrzymane pola wrażliwości wykorzystano do przebudowy tzw. rozwiązania bazowego, na rozwiązania dotyczące innych warunków chłodzenia.

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