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Numerical modelling of thermal and fluid flow phenomena in the mould channel

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Abstract

In the paper, a mathematical and a numerical model of the solidification of a cylindrical slender shaped casting, which take into account the process of filling the mould cavity with molten metal, has been proposed. Pressure and velocity fields were obtained by solving the momentum equations and the continuity equation, while the thermal fields were obtained by solving the heat conduction equation containing the convection term. Next, the numerical analysis of the solidification process of metals alloy in a cylindrical mould channel has been made. In the model one takes into account interdependence the heat transfer and fluid flow phenomena. Coupling of the thermal and fluid flow phenomena has been taken into consideration by the changes of the fluidity function and thermophysical parameters of alloy with respect to the temperature. The influence of the pressure and the temperature of metal pouring on the solid phase growth kinetics were estimated. The problem has been solved by the finite element method.

Keywords: Solidification; Molten metal flow; Mould filling; Mathematical and numerical modeling

1. Introduction

This paper concerns the modelling of the solidification process taking into account the phenomena of heat transfer and fluid flow during the initial stage of the metals allow casting process in metal moulds. During this period, the molten metal motions have an essential influence on solidification kinetics [1-4]. Therefore in this paper, an analysis of solidification kinetics, by determining the pressure, velocity and temperature fields in a system of casting-metal moulds, was made for the case of pouring from the bottom.

In recent years, only the heat conductivity equation was solved during the simulation of solidification process [5,6]. The mould cavity was assumed to be fully filled by molten metal at pouring temperature as an initial condition for computation. In this case, the influence of the metal motion on the solid phase growth kinetics during the solidification process was neglected. At the present time, the numerical simulation of casting solidification with taking into account the molten metal motion and the mould cavity filling process is often carried out. The mathematical model takes into consideration interdependence of the thermal and dynamical phenomena. Velocity fields are obtained usually by solving the Navier-Stokes equations and the continuity equation, whereas the thermal fields are calculated by solving the Fourier-Kirchhoff equation with the convection term. This is a complex and difficult problem to solve numerically [1,3,7,8]. The analysis of these phenomena is limited often only to their proceedings during the filling process of the cylindrical inlet channel [9,10] or the slender mould of fluidity test [11-13]. Just as for the case of the cavities, geometric considerations allow further simplification of the governing equations. Making assumptions relating to both the material and the geometry of the region, the general equations for continuity and momentum have been reduced to single equation for pressure. This approach leads as to further simplify of the numerical calculations [9,10,14].

In the present study, a mathematical and a numerical model of the metal solidification process in the cylindrical channels of the casting mould during its filling, has been proposed. Mathematical model takes into consideration interdependence of thermal and dynamical phenomena. Coupling of the thermal and fluid flow phenomena has been taken into consideration by the changes of the fluidity function and the thermophysical parameters of alloy with respect to the temperature. The influence of pouring parameters and rate of heat transfer on molten metal flow and its stopping moment were estimated. The whole task, that is both the heat conduction equation and the pressure equation, was solved using the finite element method [2-5,7,9].

2. Mathematical model of heat transfer during the molten metal motion

The proposed model for the numerical simulation of solidification gives consideration to the motions of metal liquid phase during the mould cavity filing process. It is based on solving the following system of differential equations in a cylindrical axisymmetry coordinate system [8,10,14]:

- the heat conduction equation containing the convection term:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)-C_{ef}\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}\right)=-\mu\Phi^2 \quad (1)$$

- the momentum equations and the continuity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v_z}{\partial r}\right)=\frac{\partial p}{\partial z} \quad (2)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rv_r\right)+\frac{\partial v_z}{\partial z}=0, \quad (3)$$

where: $T(\mathbf{x},t)$ - the temperature [K], λ - the thermal conductivity coefficient [W/(mK)], t - time [s], r – the internal radius of the channel [m], p - the pressure [N/m^2], $\mathbf{x}(r, z)$ - the coordinates of the vector of the considered node's position [m], $\mathbf{v}(v_r, v_z)$ – the velocity vector of molten metal flow [m/s], $\mu(\Theta)$ - the dynamical viscosity coefficient [Ns/m^2], $C_{ef}(T)=\rho_{LS}c_{LS}+\frac{\rho_S L}{T_L-T_S}$ - the effective heat capacity of the mushy zone [J/(m^{3}K)], L - the latent heat of solidification[J/kg], c_{LS} - the specific heat of the mushy zone [J/(kgK)], $\rho_S, \rho_L, \rho_{LS}$ - the density of solid phase, liquid phase, and mushy zone, respectively [kg/m^3], $\Phi=\frac{\partial v_z}{\partial r}$ - the shear rate [N/(sm^2)].

In the applied model of solid phase growth, the internal heat sources are not come evident in the equation of heat conductivity, because they are in the effective heat capacity of the mushy zone [3,5,8,10]. It should be noted that the above equations have a simpler form compare to general equations of a fluid flow. To simplify the equations, we employ a technique called dimensional analysis. Basically the idea is to obtain estimates of the order of magnitudes of each term in the governing equations. Terms of sufficiently low order have little influence on the numerical simulation results and so are neglected [9,10,14,15].

Further simplification is available by integrating the momentum and continuity equations. From the momentum equation (2) we see that the pressure is a function of coordinates only. For this reason it is convenient to integrate the momentum equation across the runner with the aim of obtaining expression for the z-component of velocity (v_z) as follows:

$$v_z(r)=\frac{1}{2}\frac{\partial p}{\partial z}\left[\int_0^r\frac{r'}{\mu}dr'-\int_0^{r_S}\frac{r'}{\mu}dr'\right] \quad (4)$$

where r_S is the radius indicating of melt-solid interface [m].

Integrating of the continuity equation (3) over the area of the melt channel (with respect to r) and using the definition of the average velocity (z-component) we obtain the following equation [9,10,14-16]:

$$\frac{\partial}{\partial z}\left(r_S S_1\frac{\partial p}{\partial z}\right)=0 \quad (5)$$

where we have defined

$$S_1=\frac{1}{2r_S}\int_0^{r_S}\frac{r'^3}{\mu}dr' \quad (6)$$

which is called the fluidity function for one dimensional flow. Equation (5) is a single equation for pressure that combines the momentum and continuity equation.

Finally, after making assumption regarding the material and using the effect of geometry, equations governing the flow of the molten metal in the runner of circular cross-section may take the form [10,14]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda\frac{\partial T}{\partial r}\right)+\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right)-C_{ef}\left(\frac{\partial T}{\partial t}+v_r\frac{\partial T}{\partial r}+v_z\frac{\partial T}{\partial z}\right)=-\mu\Phi^2 \quad (7)$$

$$\frac{\partial}{\partial z}\left(r_S S_1\frac{\partial p}{\partial z}\right)=0 \quad (8)$$

The system of equations (7-8) is completed by the appropriate initial conditions and the boundary conditions.

The initial conditions for pressure and temperature fields are given as [4,5,15]:

$$p(z,t_0)=p_0(z), \quad T(r,z,t_0)=T_0(r,z). \quad (9)$$

The boundary conditions specified in the considered problem were as follows [8,13-16]:

-at the inlet gate:

$$p = p_{in}\,, \quad T = T_{in}\,, \tag{10}$$

- at the mold wall:

$$v_r = v_z = 0\,, \quad \frac{\partial p}{\partial n} = 0\,, \quad \lambda_M \frac{\partial T_M}{\partial n} = -\alpha_M \left(T_M - T_a\right), \tag{11}$$

-at the flow front:

$$p = 0, \quad T = T_{in}^{*}, \tag{12}$$

- at the cavity center line:

$$\frac{\partial T}{\partial n} = \frac{\partial p}{\partial n} = 0 \qquad v_r = 0 \tag{13}$$

where: T_a - the ambient temperature [K], T_M - the mould temperature [K], α_M - the heat-transfer coefficient between mould and ambient [W/(m^{2}K)], $\boldsymbol{n}$ - the outward unit normal surface vector.

The above problem was solved by the finite element method in the weighted residuals formulation [9,14,16].

3. Example of numerical calculations

The calculations were made for the system casting-mould-ambient. The given dimensions of the essential elements of that system were as follows: d =0.017, d_M =0.067, h =0.135[m], δ =0.15[mm] [13]. The numerical calculations were made for Al-4.5% Cu alloy which poured into a cast iron mould. The thermophysical properties were taken from works [5,13] and calculations were made according to relationships shown in works [17,18]. The linear change of density (ρ) and thermal conductivity (λ) was assumed in $T_L \div T_S$ temperature interval. The variability of the dynamical viscosity coefficient (μ) with respect to temperature was determined according to Hirai's relationship. The $\mu(T)$ dependence was valid up to 0.8 value of the volume fraction of the solid phase (f_S) in the mushy zone. The viscosity values suddenly increase above 0.8 value of the f_S, and it was the reason to assume μ_S =10^5[Ns/m^2] to preserve the stability of the calculations. In this way, insignificant molten metal motion (or even a lack of it) was taken into account near the solidus line. The overheated metal with temperature T_{in} =1003[K] was poured with the pressure p_{in} =1[N/m^2] into the mould with initial temperature T_M =423[K]. The remaining characteristic temperatures were equal to: T_L =913[K], T_S =850[K] and T_a =300[K]. Thermal and flow phenomena, which proceeded in the mould cavity during filling until total solidification of the casting, were analysed (Fig.1). The influences of liquid metal movement inside the mould on the solidification kinetics of the casting were determined. Examples of calculation results are shown in the form of temperature, pressure and velocity fields.

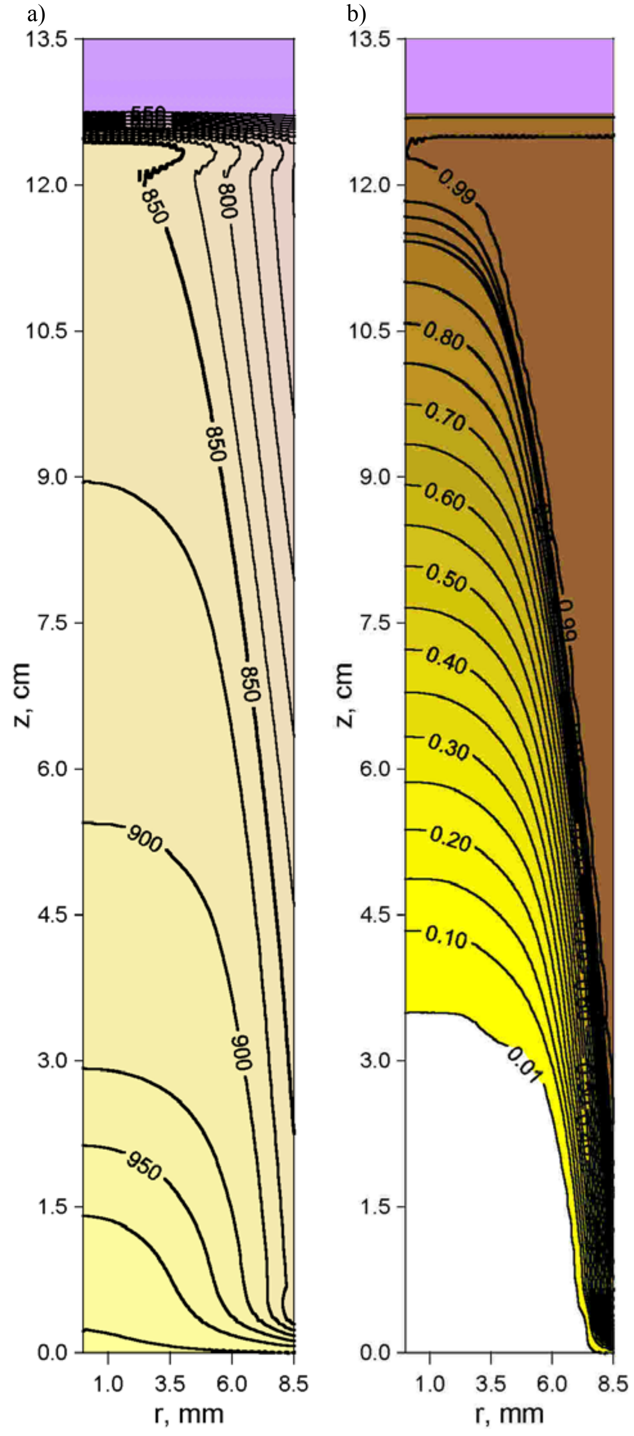

Fig.1. Temperature field (a) and solid phase fraction (b) after stopping of molten metal flow

4. Conclusions

This paper has presented the coupled model of solidification for the transient evaluation of fluid flow and heat transfer during casting solidification processes. The changes in the thermophysical parameters, with respect to temperature, were taken into consideration. The problem was solved by the finite element method. Numerical analysis included filling process of the mould cavity with molten metal, fluid flow, convectional motions of molten metal and solidification process. The influence of the pressure and the temperature of metal pouring on the solid phase growth kinetics were estimated.

References

[1] S. Mishima, J. Szekely, The modeling of fluid flow and heat transfer in mould filling, ISIJ International, vol. 29, No. 4 (1989) 324-332.

[2] G. Dhatt, D.M. Gao, A Finite element simulation of metal flow in moulds, Int. J. for Num. Meth. in Engrg., vol. 30 (1990) 821-831.

[3] R.W. Lewis, A.S. Usmani, J.T. Cross, An efficient finite element method for mould filling simulation in metal castings, Num. Meth. in Therm. Probl., vol. 7, No. 1 (1993) 273-283.

[4] L. Sowa, Model of the casting solidification taking into consideration the motion of liquid phase, Archives of Mechanical Technology and Automatization, vol. 18 (1998) 287-296 (in Polish).

[5] B. Mochnacki, J.S. Suchy, Numerical methods in computations of foundry processes, Polish Foundrymen’s Technical Association, Krakow, 1995.

[6] E. Majchrzak, M. .Jasiński, G. Kałuża, Sensitivity analysis of solidification process with respect to the geometrical parameters of casting and mould, Archives of Foundry, vol. 4, No. 14 (2004) 279-284.

[7] L. Sowa, N. Sczygiol, Computer simulation of the filling and solidification of a gravity die casting, Solidification of Metals and Alloys, vol. 2, No. 44 (2000) 329-334.

[8] R. Parkitny, L. Sowa, Numerical simulation of solidification of casting taking into account fluid flow and heat transfer phenomena. The axisymmetrical problem, Journal of Theoretical and Applied Mechanics, vol. 4, No. 39 (2001) 909-921.

[9] B.Y. Jiang, J. Zhong, B.Y. Huang, X.H. Qu, Y.M. Li, Element modelling of FEM on the pressure field in the powder injection mould filling process, Journal of Materials Processing Technology, vol. 137 (2003) 74-77.

[10] Bokota, L. Sowa, Model of thermal and fluid flow phenomena in the cylindrical inlet channel, Archives of Foundry, vol. 6, No. 22 (2006) 89-94 (in Polish).

[11] J. Campbel, Review of fluidity concepts in casting, Cast Metals, vol. 7, No. 4 (1995) 227-237.

[12] J. Mutwil, D. Niedźwiedzki, Silumins zero fluidity temperature, vol. 2, No. 4 (2004) 173-178 (in Polish).

[13] Bokota, L. Sowa, Model of the solidification of two-component alloy in the vertical fluidity test, Archives of Foundry, vol. 3, No. 10, (2003) 193-198 (in Polish).

[14] P. Kennedy, Flow analysis of injection molds, Hanser Publishers, Munich, 1995.

[15] Y.K. Shen, C.M. Ju, Y.J. Shie, H.W. Chien, Resin flow characteristics of under fill process on flip chip encapsulation, Int. Comm. Heat Mass Transfer., vol. 31, No. 8 (2004) 1075-1084.

[16] Shen, L. Wang, Q. Li, Numerical simulation of compressible flow with phase change of filling sage in injection molding, Journal of Reinforced Plastics and Composites, vol. 26, No. 4 (2007) 353-372.

[17] M. Hirai, Estimation of viscosities of liquid alloys, ISIJ International, vol. 33, No. 2 (1993) 251-258.

[18] Sichen, J. Bygden, S. Seetharaman, A model for estimation of viscosities of complex metallic and ionic melts, Metallurgical Transactions B, vol. 25B (1994) 519-525.