Application of the analysis of variance for the determination of reinforcement structure homogeneity in MMC

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Abstract

These authors propose a new definition of homogeneity verified by variance analysis. The analysis aimed at quantitative variables describing the homogeneity of reinforcement structure, i.e. surface areas, reinforcement phase diameter and percentage of reinforcement area contained in a circle within a given region. The examined composite material consisting of silicon carbide reinforcement particles in AlSi11 alloy matrix was made by mechanical mixing.

Keywords: Composite casting, Homogeneity, Analysis of variance

1. Introduction

If we assume that homogeneity as a property of composite material property means that a structure examined in various places has no statistically significant differences, then we can use statistical methods for examining material homogeneity. Examining any two-phase material such as a solid solution with pores, single-phase matrix with reinforcement, a mix of two phases etc., regardless of the degree to which the phases are refined, we can obtain – at least theoretically or using the proper tools practically – such enlargement in observations of material structure that the image will show only one of the analyzed phases. Thus, when properly magnified, a two-phase material will be perceived as a single phase material. The conclusion that follows is that each examined material will look inhomogenous when its structure is analyzed at significantly large magnification.

Therefore, one important issue in the assessment of homogeneity is the determination of the size of elemental space, in which material homogeneity is evaluated. This size is variable and mostly depends on potential application of the examined material. The present study deals with composite castings where magnification was established experimentally taking into account the size of reinforcement phase [1-2] and the need to satisfy the requirements of the ANOVA method [3] used for the analysis (number of examined objects in a given area).

Homogeneity is one of the basic parameters of a material that affects its quality. There are different characteristics of homogeneity, structural parameters used for its examination. Homogeneity characteristics include quantity (volumetric fraction), size, shape or orientation of precipitates, pores or other structural components, and in the case herein examined the structure of reinforcing particles in
suspension composites, made by the method of mechanical mixing [4-8].

Composites have a wide range of applications in many industries due to their improved desired properties, e.g. lower specific gravity, increased strength of the finished product [5, 7], enhanced thermal resistance. These materials continue to be developed, so the description of their structure is important, as composites are made by joining at least two chemically and physically different materials [4, 5, 8]. As the distribution of one material throughout the whole volume of the other has to be homogenous, the problem of inhomogeneity is important. This work is part of research into the methods facilitating the determination of metal matrix composites quality parameters. The tested material was made at the Department of Marine Materials Engineering, Maritime University of Szczecin.

2. Analysis of variance for the determination of reinforcement structure homogeneity in MMC

In the discussed case the best tool to describe homogeneity is ANOVA analysis (analysis of variance) [2-3, 9-10] as it allows to establish whether the grouping factor differentiates a given variable.

The ANOVA analysis verifies the following null hypothesis:

\[ H_0 \colon m_1 = m_2 = \ldots = m_n \]

while an alternative hypothesis reads: \( H_1 \colon \text{not all means are equal} \).

The test statistic \( F \) is calculated. If the grouping factor does not differentiate the variable, then the value \( F \) is close to 1; if \( F \) is statistically significantly larger than 1, the null hypothesis is rejected and the alternative hypothesis is accepted at the set level of test significance \( \alpha \). The test significance level \( \alpha \) is a probability that the true hypothesis will be rejected.

To examine the homogeneity of material reinforcement structure in a given area we have to identify the characteristics that describe that structure, which is a prerequisite to effective control of MMC quality. In this article, the homogeneity examination will be based on three variables:

- variable \( X \) – particle surface area \([\mu m^2]\),
- variable \( Y \) – particle diameter \([\mu m]\),
- variable \( Z \) – percentage of particle surface area contained in a circle [%].

These variables have been calculated with the Metilo program [11] for computer-aided image analysis [11] based on a model area – Fig. 1.

The grouping factor in this study is the number of one of the areas into which the examined sample was divided. The division was established at random (using a generator of random numbers) [3, 10, 12] into four and ten areas.

Fig. 1. Model area for analysis. Composite casting: reinforcement particles SiC, matrix: AlSi11, area × 600 (SEM)

The program STATISTICA PL [13] was used for tests required by the ANOVA analysis (calculated test statistics \( F \) and critical levels of test significance for each variable, for four and ten areas, respectively.

The calculations below refer to the division into four areas.

The test for the variable \( X \) – particle surface area in square micrometres:

\[ H_0 : m_{x_1} = m_{x_2} = m_{x_3} = m_{x_4} \]
\[ H_1 : \text{not all averages are equal} \]

The test statistic \( F \) value and the critical level of probability \( p \) for this variable are given in Fig 2.

![Graph showing ANOVA analysis results for particle surface area](image)

At the significance level \( \alpha = 0.05 \) there are no grounds to reject the null hypothesis on the equal means. Therefore, it can be stated [3, 9-10, 14] that for each random area the mean surface areas are equal.
The test for the variable $Y$ – particle diameter:

$$H_0 : m_{y_1} = m_{y_2} = m_{y_3} = m_{y_4}$$

$$H_1 : \text{not all averages are equal}$$

The value of test statistic $F$ and the critical level of probability $p$ for this variable are given in Fig. 3.

Fig. 3. The results of ANOVA analysis for the variable particle diameter with a division into four random areas

At the significance level $\alpha = 0.05$ there are no grounds to reject the null hypothesis on the equal means. Therefore, we can say that for each random area the mean diameters are equal.

The test for the variable $Z$ – percentage of particle surface area contained in a circle [%]:

$$H_0 : m_{z_1} = m_{z_2} = m_{z_3} = m_{z_4}$$

$$H_1 : \text{not all averages are equal}$$

The value of test statistic $F$ and the critical level of probability $p$ for this variable are given in Fig. 4.

Fig. 4. The results of ANOVA analysis for the variable percentage of particle surface area contained in a circle with a division into four random areas

At the significance level $\alpha = 0.05$ there are no grounds to reject the null hypothesis on the equal means. Therefore, we can say that for each random area the mean percentages of particle surface area contained in a circle are equal.

With the sample divided into four random areas, the analyzed variables do not show statistically significant differences, therefore the sample can be regarded as homogenous under the proposed definition. This has been also verified for the division into ten areas.

The test for the variable $X$ – particle surface area [$\mu m^2$]:

$$H_0 : m_{x_1} = m_{x_2} = m_{x_3} = m_{x_4} = m_{x_5} = m_{x_6} = m_{x_7} = m_{x_8} = m_{x_9} = m_{x_{10}}$$

$$H_1 : \text{not all averages are equal}$$

The value of test statistic $F$ and the critical level of probability $p$ for the variable $X$ are given in Fig. 5.

Fig. 5. The results of ANOVA analysis for the variable particle surface area with a division into ten random areas

At the significance level $\alpha=0.05$ there are no grounds to reject the null hypothesis on the equal means. Therefore, we can say that for each of ten random areas the mean particle surface areas are equal.

The test for the variable $Y$ – particle diameter [$\mu m$]:

$$H_0 : m_{y_1} = m_{y_2} = m_{y_3} = m_{y_4} = m_{y_5} = m_{y_6} = m_{y_7} = m_{y_8} = m_{y_9} = m_{y_{10}}$$

$$H_1 : \text{not all averages are equal}$$

The value of test statistic $F$ and the critical level of probability $p$ for the variable $Y$ are given in Fig. 6.

Fig. 6. The results of ANOVA analysis for the variable particle diameter with a division into ten random areas
At the significance level $\alpha = 0.05$ there are no grounds to reject the null hypothesis on the equal means. Therefore, we can say that for each of ten random areas the mean particle diameters are equal.

The test for the variable $Z$ – percentage of particle surface area contained in a circle [%]:

$$H_0 : m_{x_1} = m_{x_2} = m_{x_3} = m_{x_4} = m_{x_5} = m_{x_6} = m_{x_7} = m_{x_8} = m_{x_9} = m_{x_{10}}$$

$$H_1 : \text{not all averages are equal}$$

The value of test statistic $F$ and the critical level of probability $p$ for the variable $Z$ are given in Fig. 7.

![Fig. 7. The results of ANOVA analysis for the variable percentage of particle surface area contained in a circle with a division into ten random areas.](image)

At the significance level $\alpha = 0.05$ there are no grounds to reject the null hypothesis on the equal means. Therefore, we can say that for each of ten random areas the mean percentages of particle surface area contained in a circle are equal.

With the sample divided into ten random areas, the analyzed variables do not show statistically significant differences, therefore the sample can be regarded as homogenous.

3. Conclusions

When it comes to the description of the homogeneity of MMC reinforcement structure, similar examinations can feature a division into another number of areas. In the case herein considered the specific divisions matched the sample size, namely 69, as a division into a larger number of areas would not have satisfied the assumptions of the ANOVA analysis [3, 9-10, 14-16].

The presented method is efficient and effective in examining the homogeneity of material structure and is suitable for identifying homogeneous structures recognized as model ones, which may then be used for the evaluation of inhomogeneity of the other structures, which will be presented in future works of the authors.

References

[13] Program komputerowy STATISTICA PL.