ARCHIVES of FOUNDRY ENGINEERING

Published quarterly as the organ of the Foundry Commission of the Polish Academy of Sciences

ISSN (1897-3310)
Volume 10
Issue 1/2010

147 – 152

25/1

Simulation of shrinkage cavity formation during solidification of binary alloy

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Received 10.07.2009; accepted in revised form 15.07.2009

Abstract

Presented paper is focused on numerical modeling of binary alloy solidification process with connection to shrinkage cavity formation phenomenon. Appropriate matching of cooling parameters during solidification process of the cast with raiser is essential to obtain suitable properties of the manufactured part. Localization, structure and depth of the shrinkage cavity is connected to these parameters. The raiser is removed after process, so defect localization in the top part of the manufactured element is of great importance. Mathematical model of solidification process is presented in the paper. The main focus is put on the algorithm of shrinkage cavity creation process. On the basis of mathematical model the numerical approach using finite element method is proposed. On the base of mathematical and numerical model computer program is made. It is able to perform simulation of the shrinkage cavity formation in 2D region. Shape and localization of shrinkage cavity obtained from simulation is compared to defect which was created during experiment.

Keywords: Shrinkage cavity; Solidification process; Finite element method; Numerical modelling

1. Introduction

Defect of the casting is each aberration of size, weight, external appearance, internal structure, mechanical or physical properties from mandatory requirements contained in norms appropriated for particular casting. Full classification of the mechanical and physical defects occurring in castings contains about eighty types of failures [1]. The most important and usually observed defects are macroscopic, volumetric emptinesses appearing due to contraction on solidification. These defects are wide discussed in literature [2-5] and often called shrinkage cavities.

Contraction of the material goes ahead in three main stages. As the temperature reduces the first noticed contraction is that in the liquid state [5]. The volume of the liquid metal decreases linearly with falling temperature. The amount of this contraction is usually not even noticed.

The contraction on solidification occurs at the freezing point. It is caused by greater density of the solid compared to that of the liquid. Lack of material is responsible for creation of shrinkage cavities.

The last stage of contraction occurs during cooling process in the solid state. As cooling progresses and the casting tries to reduce its size it is not possible to shrink as it wishes. Significant stresses appear in particular regions of the casting. They can lead to cracking of the casting.

Presented paper considers contraction on solidification only. Numerical modelling of shrinkage cavities formation is not often discussed in literature. Examples of papers considering that phenomenon are articles [6-10].

2. Mathematical model

Considered region consists of few subregions which are identified by content of liquid solid and gaseous material (fig. 1). Most important domains in solidifying casting are solid region Ω_S, liquid region Ω_L and air filled region Ω_A respectively. Additionaly mixture of mentioned phases may appear. During solidification solid-liquid area Ω_{S+L} is observed as well as mixture of solid and gas Ω_{S+A} or compound of three of them Ω_{S+L+A}. Mixture of liquid and gas is also allowed within presented model, however it appears temporarily in the thin layer when cooling rate on the top boundary is low. Top level of the liquid decreases due to progress of solidification process near the walls. Lack of liquid is compensated by air which appears in the thin layer near the top boundary. Because of small size of that region it is neglected in the figure.

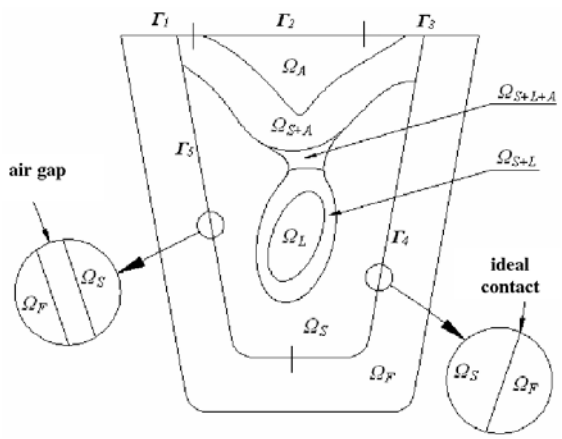

Fig. 1. Mould and casting consisted of regions filled with mixture of the solid, liquid and gas

The governing equation for the model is equation of transient heat diffusion in two dimensional region. The emission of latent heat during solidification is built in equation as effective heat capacity [11]:

$$\left(\lambda T,_i\right),_i = c_{ef}\rho\frac{\partial T}{\partial t} \tag{1}$$

where T [K] denotes temperature, t [s] - time, λ [W/(mK)] – coefficient of thermal conductivity, ρ [kg/m^3] – density, c_{ef} [J/(kgK)] – effective heat capacity.

Thermal conductivity, density and specific heat are functions of solid, liquid and gas content. Solid content f_s is calculated according to following formula:

$$f_S = \begin{cases} 1, & \text{if} \quad T < T_S \\ \dfrac{T_L - T}{T_L - T_S}, & \text{if} \quad T_S \le T \le T_L \\ 0, & \text{if} \quad T > T_L \end{cases} \tag{2}$$

Mentioned parameters also indirectly depend on temperature. Equations (3) show the way for averaging these quantities. On the other hand, the way for calculation of particular phase content is important part of the shrinkage cavity formation algorithm.

$$\begin{aligned} \lambda &= f_s\lambda_s + f_l\lambda_l + f_a\lambda_a \\ \rho &= f_s\rho_s + f_l\rho_l + f_a\rho_a \\ c &= f_s c_s + f_l c_l + f_a c_a \end{aligned} \tag{3}$$

where c [J/(kgK)] is specific heat, s, l, a indices denote solid, liquid and gas respectively.

Effective heat capacity is used for modelling of latent heat emission during solidification. When the temperature in the casting is between solidus T_S and liquidus T_L specific heat is augmented by a term called spectral heat of solidification. That modification is showed below:

$$c_{ef} = c + \frac{L}{T_L - T_S} \tag{4}$$

where L [J/kg] denotes latent heat of solidification.

Equation (1) is supplemented by appropriate boundary conditions

$$\mathbf{x} \in \Gamma_{1-5} : -\lambda\ \mathbf{n}\cdot\mathbf{grad}T = \alpha\,(T - T_\infty) \tag{5}$$

and initial condition

$$t = 0 : T\big|_{\Omega_L} = T_0 \tag{6}$$

where α [W/(m^{2}K)] is heat convection coefficient, T_∞ [K] – ambient temperature, T_0 [K] – initial temperature of the liquid alloy, $\mathbf{n}$ – vector normal to the casting external boundaries.

3. Numerical model

Weighted residual method [12, 13] is used for transformation of heat diffusion equation to the weak form. Equation (1) is multiplied by weighting function w and integrated over region Ω:

$$\int_\Omega w\left[\left(\lambda T,_i\right),_i - c_{ef}\rho\frac{\partial T}{\partial t}\right]d\Omega = 0 \tag{7}$$

The weak form is obtained by decreasing order of equation (7) with using of Green's theorem:

$$\int_\Omega \lambda w,_i T,_i\, d\Omega + \int_\Omega c_{ef}\rho w\frac{\partial T}{\partial t}d\Omega = -\oint_\Gamma w q_n d\Gamma \tag{8}$$

Weak form (8) is discretized over space with using of standard Bubnov-Galerkin method. After time discretization procedure (Euler backward scheme) in relation to time derivative and agregation of the discrete model, global finite element equation is obtained [13]:

$$\left(\mathbf{K} + \frac{1}{\Delta t}\mathbf{M}\right)\mathbf{T}^{f+1} = \frac{1}{\Delta t}\mathbf{M}\mathbf{T}^{f} + \mathbf{B} \tag{9}$$

where **K** denotes heat conductivity matrix, **M** – heat capacity matrix, **B** – right hand side vector.

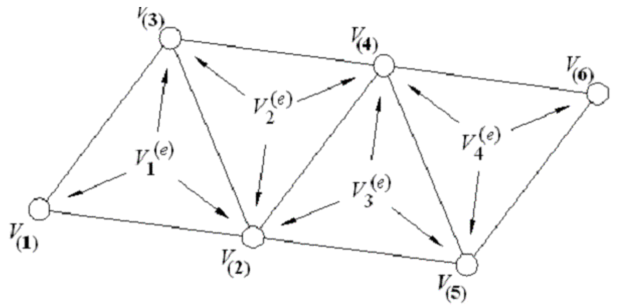

Fig. 2. Recalculation of elemental volumes to nodal volumes

Algorithm of shrinkage cavity calculation requires sorted list of nodes which have non-zero liquid content. They should be sorted according to decreasing vertical (*y*) coordinate. It means that nodes located at the top of the casting are in the first positions in the sorted list. In the next step nodal volumes are calculated for all nodes in the finite mesh (fig. 2). Analyzed domain is initially filled with hot liquid material which volume is equal to total volume of casting *V*. It denotes that

$$\sum_{i=1}^{n} V_{l(i)} = V, \quad \sum_{i=1}^{n} V_{s(i)} = 0, \quad \sum_{i=1}^{n} V_{a(i)} = 0 \tag{10}$$

where *n* is total number of nodes, $V_{l(i)}$, $V_{s(i)}$, $V_{a(i)}$ [m^3] – volume of liquid, solid and air in the ith node.

During cooling process of the casting temperature is monitored in the nodes with non zero liquid content at each time step. If real temperature *T* in any node decreases below liquidus temperature T_L the procedure of introducing an air into casting is activated. Known value of contraction on solidification S_h allows to calculate progression of shrinkage cavity. On the base of known f_s at the moment *t* (previous step) and $t + \Delta t$ (current time step) volumetric increment of the solid phase in the ith node is calculated in the following way:

$$\Delta V_{s(i)} = \left(f_{s(i)}(t + \Delta t) - f_{s(i)}(t)\right) V_{l(i)}(t) \tag{11}$$

Actual volume of the solid phase in the ith node is calculated using below equation

$$V_{s(i)}(t + \Delta t) = V_{s(i)}(t) + \Delta V_{s(i)} \tag{12}$$

Total volumetric increment of the solid phase in the casting is obtained as a sum of nodal increments:

$$\Delta V_s = \sum_{i=1}^{n} \Delta V_{s(i)} \tag{13}$$

Volume of liquid phase in the ith node is actualized according to below formula

$$V_{l(i)}(t + \Delta t) = V_{l(i)}(t) - \Delta V_{s(i)} \tag{14}$$

Corrected content of the solid phase in each node is obtained as quotient of $V_{s(i)}$ divided by nodal volume $V_{(i)}$:

$$f_{s(i)}(t + \Delta t) = \frac{V_{s(i)}(t + \Delta t)}{V_{(i)}} \tag{15}$$

Global increment of air volume at current time step is determined as a result of multiplication ΔV_s by shrink coefficient S_h:

$$\Delta V_a = \Delta V_s S_h \tag{16}$$

Algorithm of calculation of nodal air contents is realized in the following way:

1. $i = 1$,
2. check if $\Delta V_a - V_{l(i)}(t + \Delta t) > 0$,

if condition in point 2 is fulfilled, following operations are realized:

2.1. $\Delta V_a = \Delta V_a - V_{l(i)}(t + \Delta t)$,
2.2. $V_{a(i)}(t + \Delta t) = V_{a(i)}(t + \Delta t) + V_{l(i)}(t + \Delta t)$,
2.3. $V_{l(i)}(t + \Delta t) = 0$,
2.4. $f_{l(i)}(t + \Delta t) = 0$,
2.5. $f_{a(i)}(t + \Delta t) = V_{a(i)}(t + \Delta t) / V_{(i)}$,

if condition in point 2 is not fulfilled, following operations are realized:

2.6. $V_{a(i)}(t + \Delta t) = V_{a(i)}(t + \Delta t) + \Delta V_a$,
2.7. $V_{l(i)}(t + \Delta t) = V_{l(i)}(t + \Delta t) - \Delta V_a$,
2.8. $\Delta V_a = 0$,
2.9. $f_{l(i)}(t + \Delta t) = V_{l(i)}(t + \Delta t) / V_{(i)}$,
2.10. $f_{a(i)}(t + \Delta t) = V_{a(i)}(t + \Delta t) / V_{(i)}$,

3. $i = i + 1$,
4. if $\Delta V_a > 0$ return to point 2, else procedure is finished ant the next time step is analyzed.

Procedure showed in points 1–4 is executed at each time step causing gradual incrementation of air volume in the casting. Shrinkage cavity appears as the consequence of that process.

4. Example of calculations

Geometry of the casting with raiser used in calculations is showed in the fig. 3. The model was built on the base of casting made during the experiment[1].

Region of the mould Ω_F is omitted during calculations, however its influence on the cooling process is taken into account by appropriate boundary conditions.

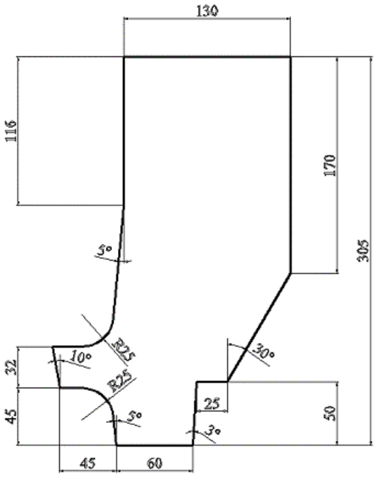

Fig. 3. Geometry of the casting with raiser (size in milimeters)

Newton boundary conditions were assigned to all boundaries with ambient temperature T_∞=300 [K]. Except the top boundary heat convection coefficient on the walls was equal to α=100 [W/(m^{2}K)]. Initial temperature was equal to 1800 [K].
Two variants of calculations were made. Coefficient α=10 [W/(m^{2}K)] was taken in the first case and α=200 [W/(m^{2}K)] in the second one.

Material properties of steel with 0.55 % of carbon as well as parameters of air are showed in the tab. 1.

Table 1.
Material properties used in the simulation

Material property	Liquid phase	Solid phase	Air
λ [W/(mK)]	23	35	0.027
ρ [kg/m^3]	6915	7800	1.1
c [J/(kgK)]	837	644	1000
Parameter of solidification	Value		
L [J/kg]	270000		
T_L [K]	1766		
T_S [K]	1701		
S_h [-]	0.0575		

Surface of the casting was discretized into 73136 triangular finite elements with total number of nodes equal to 37025. Both variants of calculations were made with constant time step Δt=0.25 [s], until entire material became solid.

[1] Experiment for KBN project no. 7 TO7 00917, Polish Foundrymen's Technical Association, AGH, Cracow, 2002.

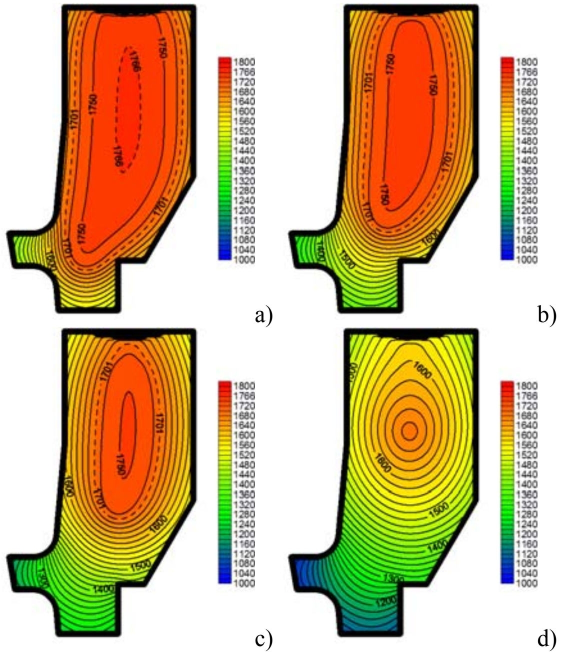

Fig. 4. Temperature of the casting according to low cooling rate on the top boundary after a) 500 [s], b) 750 [s], c) 1000[s], d) 1350 [s]

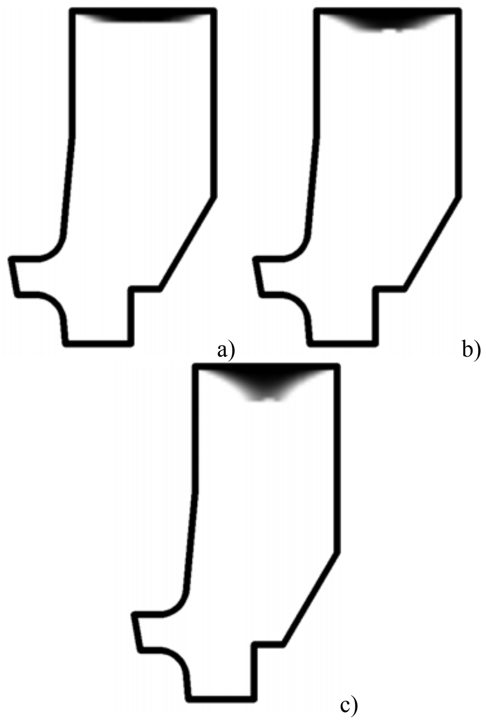

Fig. 5. Shape of the shrinkage cavity according to low cooling rate on the top boundary after a) 500 [s], b) 750 [s], c) 1000[s]

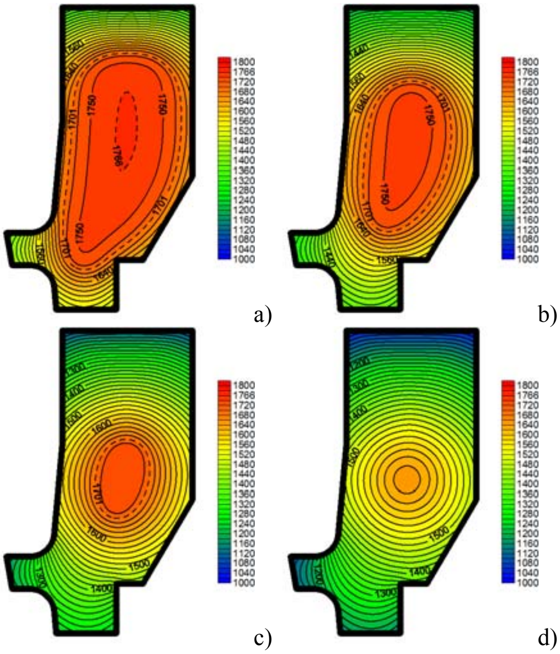

Fig. 6. Temperature of the casting according to high cooling rate on the top boundary after a) 500 [s], b) 750 [s], c) 1000[s], d) 1175 [s]

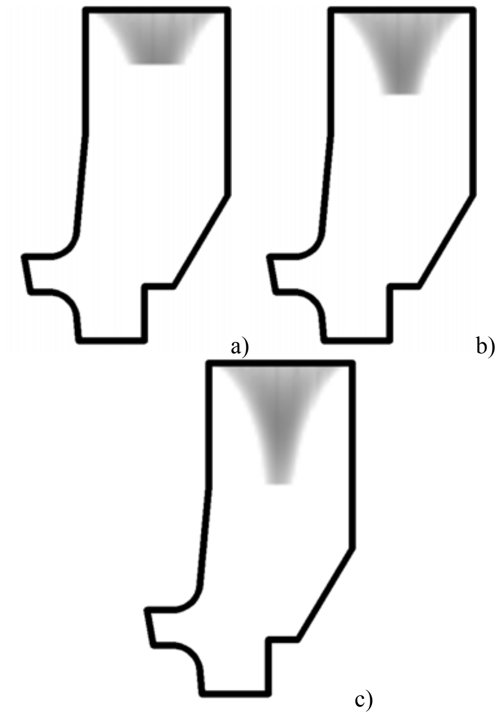

Fig. 7. Shape of the shrinkage cavity according to high cooling rate on the top boundary after a) 500 [s], b) 750 [s], c) 1000[s]

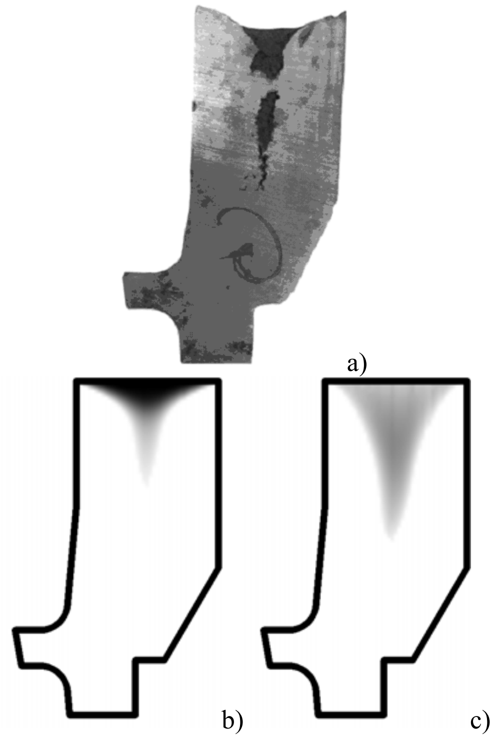

Fig. 8. Shape and localization of the shrinkage cavity a) real defect, b) simulation with low cooling rate on the top boundary, c) simulation with high cooling rate on the top boundary

Temperature change according to time is presented in the figs. 4 and 6. Differences between temperature obtained for low and high cooling rates on the top boundary are clearly visible. If the cooling rate is low horizontal temperature gradient dominates and position of the thermal centre is located shallow or else vertical one is distinct and hottest zone moves deeper. Position of the thermal centre affects shrinkage cavity vertical size.

Air distribution in the casting is showed in the figs. 5 and 7. Volumetric contraction of the material during solidification causes air penetration into the casting. If the raiser is cooled slowly final shrinkage cavity looks like a shallow cone (figs. 5a-c), but if the cooling rate is high, defect is slightly deeper (figs. 7a-c). In both cases bottom part of the shrinkage cavity lies near the thermal centre of the casting. From the practical point of view it is necessary to choose appropriate cooling conditions because shallow localization of thermal centre is desired. It can be obtained by introducing thermal isolation of the raiser or boosting cooling rates in the bottom of the casting.

In the fig. 8a structure and localization of the shrinkage cavity in the real casting is presented. Defect consists of two non connected areas with solid material between them. Results of computer simulation shows only one defect in both variants of cooling. Real casting is a three-dimensional body, while simulation was executed in two-dimensional region. The algorithm of shrinkage cavity calculation prevents from formation of disconnected defects too. Real shrinkage cavity is comparable to that formed under conditions of high cooling rate of the top raiser boundary (fig. 8c).

5. Conclusions

Presented numerical model and its computer implementation are able to predict localization and the shape of shrinkage cavities which often appear during casting process. Appropriate choice of cooling parameters in connection with analysis of results obtained from computer simulation may be helpful in selection of optimal process parameters. Suitable control of the casting process will result in shallow localization and regular shape of the potential defect. This is desired from the technological point of view.

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