Modeling of mould cavity filling process with cast iron in Lost Foam method
Part 3. Mathematical model – pressure inside the gas gap

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Abstract

In this work mathematical model describing changes of pressure inside the gas gap was shown during manufacturing gray cast iron castings with use of lost foam process. Authors analyzed the results of numerical simulation enclosing influence of foamed polystyrene pattern density, permeability and thickness of refractory coating on pressure changes in the gap. Studies have shown, that all these parameters have significant influence on pressure inside the gas gap.

Keywords: Metal casting, Lost foam process, Foamed polystyrene pattern

1. Introduction

In lost foam process, the foamed polystyrene patterns are used, on which refractory coating is applied [1, 2]. This coating creates a working surface of the mould and prevents the liquid metal to penetrate to the dry sand bed [3]. During filling the mould cavity with liquid metal, a great amount of gases occurs due to pattern gasification and results in pressure increase in gas gap, what slows the metal flow down. Refractory coating needs to enable removal of gas products of polymer decomposition outside the mould to lower the pressure to value, which enables mould filling with assumed metal flow rate. It can be assumed, that the pressure is influenced mainly by refractory coating parameters – its permeability and thickness. To low permeability causes increase in the pressure, what can result in waster casting [4], coating fracture and liquid metal outflow. In such case on casting surface veining occurs. Too high pressure disables proper mould filling and misrun castings can occur.

2. Mathematical model

2.1. Assumptions

Mathematical model was developed with following assumptions:
- gases removal from the gap proceeds through the coating applied to the pattern and dry sand bed (casting mould),
- gas filtration proceeds through the surface limited by gas gap height.

Physical model of gas gap was shown and described in part 1 of this work.

2.2. Gas pressure inside the gap

For determination of gas pressure changes the gas state equation was used:
\[ P_g \cdot V_g^k = n \cdot R \cdot T_g \]  
(1)

where:
- \( P_g \) - gas pressure in the gap, Pa
- \( V_g \) - gas gap volume, \( m^3 \)
- \( n \) - number of moles of formed gases, mol
- \( R \) - universal gas constant, \( \frac{J}{mol \cdot K} \)
- \( T_g \) - temperature of gases, K
- \( k \) - dimensionless adiabate exponent.

In the equation of gas state there is number of moles of gas \( n \), which needs to be taken into account to determine the molar mass of gases inside the gap. This task is difficult, because composition of polystyrene gasification products is not known. Work [5] provides information, that at temperature of 500 °C volatile products consists mainly styrene monomer \( C_8H_8 \). With growing temperature monomer particle undergoes another decomposition to light hydrocarbons such as: \( C_2H_4 \), \( C_3H_6 \), \( C_4H_8 \) and \( C_2H_2 \). At temperature above 1000 °C, thermal decomposition of polystyrene results in formation of significant amount of carbon in form of graphite and even occurrence of hydrogen. Attempts been made [5] to determine the composition of gasification products but obtained results were not satisfactory. In works [6, 7] a volume of gases produced from mass unit of polystyrene was determined in function of temperature and obtained values were 250 cm\(^3\)/g and 800 cm\(^3\)/g for temperature 750 and 1300°C, respectively. This data enabled moles number produced during styrene gasification. At temperature of 1300°C (pouring temperature for gray cast iron), number of moles was equal to 3.5. As a confirmation of this value a analysis of styrene monomer particle can be quoted. At this temperature such particle can decompose according to following equations:

- \( C_8H_8 \rightarrow C_6H_6 + 2C_2H_2 \)
- \( C_8H_8 \rightarrow C_3H_6 + C \)
- \( C_8H_8 \rightarrow 4C_2H_2 \)
- \( C_8H_8 \rightarrow 2C_2H_4 + 4C \)
- \( C_8H_8 \rightarrow C_6H_4 + C_2H_2 + 2C + H_2 \)

Thus, from 1 mole of styrene 2 to 6 moles of gas can be obtained.

Using real gas constant \( R_g = \frac{R}{m_{mol}} \) in gas state equation, it can be written as follows:

\[ p_g \cdot V_g = m_g \cdot R_g \cdot T_g \]  
(2)

where:
- \( m_g \) - weight of gases in the gap, kg

In this equation pressure of gases \( p_g \), gap volume \( V_g \) and weight of gases in the gap \( m_g \) are time dependent variables. To obtain differential equation for pressure changes inside the gap, the first derivative after time needs to be calculated, with respect to the fact, that the left side of the equation is the product of two functions:

\[ \frac{dP}{d\tau} = \frac{R_g \cdot T_g}{m_g} \cdot \frac{dm_g}{d\tau} \]  
(3)

After substitution simultaneously for gap volume:

\[ V_g = (y_{mod} - y_{net}) \cdot A_{mod}, \]  
the differential equation describing pressure changes in time was obtained, as follows:

\[ \frac{dP}{d\tau} = \frac{dm_g}{d\tau} \frac{R_g \cdot T_g}{m_g} \]  
(4)

Gases weight in the gap can be expressed as a difference between gases weight produced during pattern gasification and gases filtrated through the coating and the sand bed:

\[ \frac{dm_g}{d\tau} = \frac{c_{mod} \cdot A_{mod} - m_{filtr}}{\tau} \]  
(5)

Gases weight filtrated \( m_{filtr} \) can be calculated with use of Darcy’s law for porous medium permeability, which in foundry conditions takes the form of [8]:

\[ K = \frac{V_{filtr} \cdot L_{filtr}}{A_{filtr} \cdot \tau \cdot \Delta p} \]  
(6)

where:
- \( V_{filtr} \) - filtered gases volume, \( m^3 \)
- \( L_{filtr} \) - filtration distance, m
- \( A_{filtr} \) - area, through which the gases are filtrated, \( m^2 \)
- \( \Delta p \) - pressure drop on distance \( L_{filtr} \), Pa

After substituting to equation (6) following relations:

\[ V_{filtr} = \frac{m_{filtr}}{p_g} \]  
(7)

\[ L_{filtr} = \frac{1}{K_{pok}} \cdot \frac{L_{piaxu}}{K_{piaxu}} \]  
(8)

\[ \Delta p = P_g - P_f \]  
(9)

where: \( P_f \) pressure inside the mould, Pa
- \( A_{filtr} = O_{filtr} \cdot (y_{mod} - y_{net}) \)  
(10)

where: \( O_{filtr} \) - foamed polystyrene pattern perimeter
- \( O_{filtr} = 2 \cdot (a_{mod} + b_{mod}), m \)

the relation of mass filtration rate was obtained:
3. Numerical simulation studies

3.1. Range of numerical simulation studies

Numerical simulation studies was conducted for pattern with geometry shown in fig. 2. of part 1 of the work. Conducted studies enclosed changing parameters: coating permeability in range of $K_{pok} = 1 \div 9.5 \cdot 10^{-9} \, m^2/Pa\cdot s$, coating thickness in range of $L_{pok} = 0.3 \div 1.5 \, mm$ and polystyrene pattern density in range of $\rho_{mod} = 10 \div 40 \, kg/m^3$ and their influence on pressure changes inside the gap. Moreover, for numerical simulation additional parameters were assumed: cast iron density $\rho_{met} = 7200 \, kg/m^3$, dry sand permeability $K_{piasku} = 8.5 \cdot 10^{-6} \, m^2/Pa\cdot s$, sand bed thickness $L_{piasku} = 15 \, cm$, temperature of liquid cast iron $T_{met} = 1573 \, K$, pressure inside the mould $P_{f} = 100 \, kPa$ and thermo-physical properties of polystyrene described in part 1.

3.2. Analysis of numerical simulation results

One of the most important parameters influencing the process of mould cavity filling is the permeability of refractory coating applied on foamed polystyrene pattern. In fig. 1 and 2 influence of coating permeability on gas pressure in the gap was shown. It can be seen, that for smaller permeability of the coating, the pressure inside the gap is higher, because the filtration of gases is much more difficult in such case and proceeds at higher pressure values. For assumed lowest values of permeability the pressure inside the gap reaches high value of 132 kPa in the initial phase of the process. This value can be sufficient to cause the back flow of poured metal, in case of lower permeability of the coating, the pressure inside the gap is higher, because the filtration of gases is much more difficult in such case and proceeds at higher pressure values. For assumed lowest values of permeability the pressure inside the gap reaches high value of 132 kPa in the initial phase of the process. This value can be sufficient to cause the back flow of poured metal, in case of smaller height of the main gate.

![Graph](image-url)

**Fig. 1.** Changes in pressure inside the gap for different values of refractory coating permeability $K_{pok}$.
Permeability of refractory coating, $10^{-9}$ m²/(Pa·s)

Mean pressure in gas gap, kPa

In fig. 3 and 4 influence of coating thickness on pressure inside the gap is shown. Increase of coating thickness causes increase in pressure inside the gap, because longer distance of filtration results in higher flow resistance and the removal of gases is more difficult. Analyzing diagrams in fig 1 and 3, one can conclude, that to decrease the pressure inside the gap it is needed to apply coating with higher permeability. Because of technological reasons it is difficult to obtain coatings with thickness smaller than 0.5 mm, with proper resistance to cracking and fracture – which decreases with the coating thickness.

Fig. 3. Changes of pressure inside the gap for different values of refractory coating thickness $L_{pok}$

Influence of pattern density on pressure inside the gap is shown in fig. 5 and 6. Pressure increases together with increase in pattern density, because in the same volume of the pattern the polystyrene amount is higher and during gasification higher amount of gases is produced. Maximum pressure at the beginning of mould cavity filling reaches values about 125 kPa to 128 kPa, for density 10 and 40 kg/m³, respectively and drops in the next stage, gradually, as the difference between the metal column inside the mould and the main gate decreases.

Fig. 5. Changes in pressure inside the gap for different values of pattern density $\rho_{mod}$

4. Summary

Pressure inside the gas gap is one of the most important factors influencing filling of the mould cavity in lost foam process. Presented results of numerical simulation enabled determination of pressure changes due to technological parameters of the process. Conducted analyses showed, that during filling the mould cavity with liquid cast iron pressure in the gas gap reached value of 124÷132 kPa at the beginning of the filling process and then decreased to value of 115÷116 kPa. Described values are significant in respect to aluminum alloys casting, for which obtained pressure were 106÷109 kPa and 103÷104 kPa, at the beginning and the end, respectively [9]. Pressure in the gap can be decreased by changes in refractory coating parameters, that is by decreasing the coating thickness or increasing its permeability or using patterns with lower density.

Studies described in this work (part 1÷3) confirmed, that coating permeability and thickness as well as pattern density were essential factors in lost foam process. Presented results allow...
proper selection of technological parameters resulting in elimination of flaws from casting manufacturing with use of lost foam process.

References