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Geometrical modulus of a casting and its influence on solidification process

F. Havlicek, T. Elbel*
VSB – Technical University of Ostrava, 17. listopadu 15, CZ 708 33 Ostrava-Poruba,
Department of Foundry Engineering, FMMI
*Corresponding author. E-mail address: tomas.elbel@vsb.cz

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Abstract

Object: The work analyses the importance of the known criterion for evaluating the controlled solidification of castings, so called geometrical modulus defined by N. Chvorinov as the first one. Geometrical modulus influences the solidification process. The modulus has such specificity that during the process of casting formation it is not a constant but its initial value decreases with the solidification progress because the remaining melt volume can decrease faster than its cooling surface.
Methodology: The modulus is determined by a simple calculation from the ratio of the casting volume after pouring the metal in the mould to the cooled mould surface. The solidified metal volume and the cooled surface too are changed during solidification. That calculation is much more complicated. Results were checked up experimentally by measuring the temperatures in the cross-section of heavy steel castings during cooling them.
Results: The given experimental results have completed the original theoretical calculations by Chvorinov and recent researches done with use of numerical calculations. The contribution explains how the geometrical modulus together with the thermal process in the casting causes the higher solidification rate in the axial part of the casting cross-section and shortening of solidification time.
Practical implications: Change of the geometrical modulus negatively affects the casting internal quality. Melt feeding by capillary filtration in the dendritic network in the casting central part decreases and in such a way the shrinkage porosity volume increases. State of stress character in the casting is changed too and it increases.

Keywords: Modulus of casting, Solidification rate, Casting quality

1. Introduction

The term modulus is a general notion expressing a comparable unit of mutual geometrical, physical or technical quantities that determinate the course of the given process. The work deals with the solidification process of castings of different geometry and the determining quantities are the melt volume and its cooling surface in mutual relation what has been used for the first time by N.Chvorinov [1] and he designated it as size ratio R:

$$R = V/A, \tag{1}$$

where V is the melt volume and A is surface area.

Later on this relation starts to be designated as casting modulus $\mathbf{M_C}$. With the aid of the modulus square the time of casting solidification can be expressed. The modulus also explains the differences in conditions of solidification time of basic geometrical bodies of a plate, a cylinder (a quadratic prism), and a sphere (a cube).
Even at the beginning of 21^{st} century, in the time of numerical modelling and simulation foundry programmes, Nikolai Chvorinov is quoted in many journal articles and monographs on solidification and in textbooks. Let us to name P.R. Sahm and his colleagues [2], D.M. Stefanescu [3], J. Campbell [4], F.P. Schlegg [5] J.A. Dantzig and M. Rappaz [6]. The last named authors have stated that Chvorinov’s rule is used in daily foundry practice to

estimate the solidification time for parts. It can be further extended to assess the solidification pattern in parts with varying section sizes, and thus predict the overall progress of solidification through the part. Casting designers can then place risers appropriately to ensure that the last regions to freeze, where shrinkage porosity is likely, are situated outside the final product. In many cases, this simple calculation is sufficient for designing an acceptable casting. However, many castings are either too complicated, or require too much precision, for this approximate approach to be adequate, and in these cases computer simulations of the filling and heat transfer are carried out [6]. Knowledge results from our previous works [7, 8] that the modulus value of the cylinder and sphere castings is changing during solidification process. It is caused by irregular change of the melt volume V and the cooled surface A. It influences the increase of the solidification rate in the internal casting part and in such a way also the stress condition changes, limited melt feeding in the solidification zone in the axial region, and the shortening of solidification time. Another physical influence – the change of the casting temperature regime during solidification – acts in the same respect.
Based on those experimental results of casting solidification and their theoretical analysis the total solidification time is divided in the first and second solidification phase. In each phase with solidification time a contrary temperature regime, solidification rate, stress condition, and casting defects are developed.

2. Determination of the casting modulus of basic constructional bodies

Due to study it has been found out that castings of different design, a cylinder, a sphere, will solidify (under specific preconditions) with the same modulus sooner than the plate casting.
From the point of view of iterm accuracy, when the M_C modulus expresses solidification conditions of geometrically different bodies, the term „relative geometrical casting modulus“ is more accurate. Besides the geometrical modulus a term thermal modulus is used which is a modulus of a riser with solidification time corresponding to a riser with solidification time prolonged due to the influence of thermal insulation (decreased value of thermal diffusivity coefficient of the mould b_f).
When comparing solidification time of the plate, cylinder, and ball casting it is necessary to issue from the real efficient (effective) cooled casting surface that influences solidification time from the surface normal to the cooled surface with no influence on heat removal by side walls. Analytical calculations of solidification times by Chvorinov [9], Halbart [10] and other authors therefore resulted from a precondition that the plate and the cylinder have semi-infinite dimensions and therefore the peripheral and face surfaces are not considered in the calculation. For research and technological purposes some authors checked the lowest ratio of the side length of the plate casting L_p and the cylinder (prism) length $L_{c,pr}$ to the plate thickness x_p or to diameter D_c of the cylinder during solidification 5:1. It means that in this case the side and face walls can be neglected as they don’t thermally influence solidification of the given casting in the centre of its length. . This supposition confirms our recent study of numerical simulation of solidifying cylinders with different ratio L/D from 3 to 10:1. Calculations of the geometrical modulus for the infinite height and length of basic bodies are given in the first line of table 1.
Calculation of the casting moduli with considering the total cooled body surface was done by T. Elbel [11]. He issued from consideration that the characteristic dimensions of the plate, cylinder, prism, case castings are the n multiples of their characteristic dimension (thickness, diameter, side). If this multiple achieves the infinity then the formulae in the first line of table I are valid for modulus calculation. E.g. for calculating the cylinder modulus M_C a relation is valid as follows:

$$M_C = \frac{r}{2} \cdot \frac{n}{n+1} = \frac{r}{2}.N_C \tag{2}$$

in which the r is the cylinder radius, the n gives by how much is the cylinder length higher (the multiple) than the radius. Similar appreciations was treated by W.Longa [12] too. Values of multipliers for individual bodies are given in second line of table 1. For the cube and the sphere the multiplier equals to one as the ball diameter and the cube edge unambiguously determine both the body surface and its volume.

Table 1.
Calculation of the modulus of basic bodies for semi-infinite height or length of the body and multiplier values of those formulas for real body dimensions.

Calculation	Plate	Sleeve	Cylin-der	Prismee	Sphere	Cube
$M_c = \frac{V}{A}$	$\frac{X_p}{2}$	$\frac{X_{sl}}{2}$	$\frac{D_c}{4}$	$\frac{X_{pr}}{4}$	$\frac{D_s}{6}$	$\frac{X_c}{6}$
Multiplier N	$\frac{n}{n+2}$	$\frac{n}{n+1}$	$\frac{n}{n+1}$	$\frac{n}{n+0}$	1	1

the n ….. multiple of basic dimensions (the body thickness, width, radius)

It is evident from the table 1 that the plate and sleeve (bushing), cylinder and prism, sphere and cube casting couples have the same value of the casting modulus and also mutually coincident conditions of cooling during solidification and thus the corresponding solidification time too.
The plate casting solidifies from two sides by cooling of parallel mould surfaces towards the thermal axis and the size of both cooled surfaces is constant for solidification time. Therefore the modulus value of the plate corresponds to a half of the plate thickness $x_p/2$ and it is not changed for the whole solidification time.
The cylinder modulus is derived from the ratio of the melt volume and the cooled cylinder surface (or more simply the cross-section surface divided with its perimeter) with the result of $D_C/4$. At the same time the sphere modulus is a result of the ratio of the volume and its surface and it has a value of $D_S/6$. For the cylinder and the sphere the moduli are changed during the solidification

process. It is caused by irregular change of the melt volume V and of the cooled surface A. The volume is reduced more quickly than the cooled surface (solidus). It leads to increasing the solidification rate (Figure 1) and shortening the solidification time of the cylinder and ball castings (Figure 2). So keeping line with a later work by Chvorinov [13] the result for the solidification end is for the cylinder 0.05 and for the sphere 0.33 from the relative solidification time of the plate $\tau / M^2 = 1$.

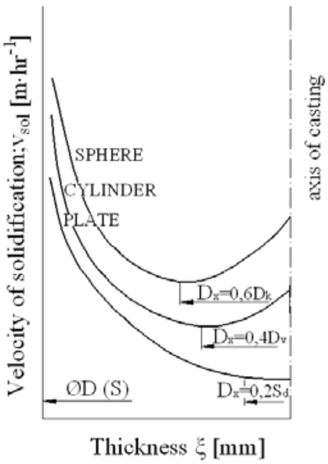

Fig. 1. Comparison of calculated solidification rate in cross section of basic shape castings

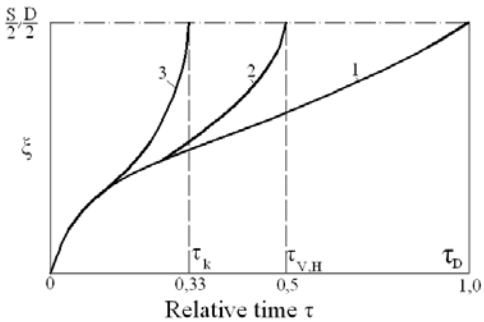

Fig. 2. Characteristic curves for isosolidus progression of basic shape castings

Asymmetric growth of solid thickness and solidified volume of cylinder shape castings in comparison with plate casting elaborated w.Longa in an anterior monograph [14].

3. Influence of the casting modulus on the solidification process and formation of defects in castings

Geometrical modulus of the casting introduced as a first criterion of internal quality control of castings is of a great importance for foundry industry even in the time of use of simulation programmes. J. Campbell [4] has stated that the rightness of the Chvorinov's rule has already been many times successfully proved. The modulus method provides a general process for solving the casting feeding for ensuring its healthiness. Technological use of the casting modulus consists in e.g. its including in calculations of the riser sizes [15]. R.W. Ruddle [16] in a paper on the risering history from 1971 highly appreciated the importance of the Chvorinov's work and he stated as follows: „it was unlucky that the Chvorinov's work was so closely considered identical with his rule that other aspects of his work aimed at practical risering of castings were ignored".
The modulus method of calculating the riser sizes consists in graduated value of the casting modulus M_C, the riser neck M_N and the proper riser M_R.

E.g. : $M_C = 1 < M_N = 1{,}1M_C < M_R = 1{,}2M_C$ (3)

In the theoretical field the modulus requires next research. Together with the thermal process in the casting it causes the higher solidification rate in the axis part of the cross-section and shortening of solidification time. But it has negative impacts for casting internal quality. Thus the melt feeding by capillary filtration (penetration in the central zone) is reduced and in such a way the shrinkage porosity volume increases. State of stress character in the casting is changed too and it increases.
Solidification process of the plate as a basic geometrical shape is expressed by a parabolic relation (4) and the curve 1 in Figure 3 as follows :

$$\xi = k \cdot \sqrt{\tau} \; ; \quad [\mathrm{m}] \qquad (4)$$

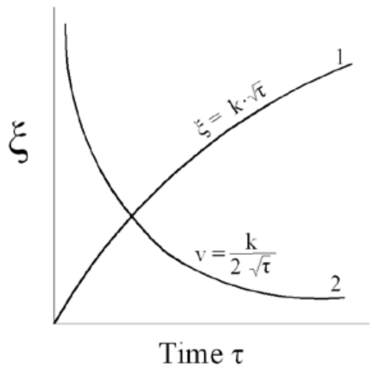

Fig. 3. The thickness of solidified layer of metal and solidification velocity

With derivation of the parabolic curve the instantaneous velocity of solidification in casting cross-section is obtained curve 2 in Figure 3 as follows:

$$v = \frac{d\xi}{d\tau} = \frac{k}{2\sqrt{\tau}} \qquad [\mathrm{m.. hr^{-1}}], \qquad (5)$$

where:
ξ – thickness of the solidified layer [m]
k – solidification coefficient [m . h $^{-1/2}$]
τ – time [hr]

Solidification time is given from it by a relation as follows:

$$\tau = \left(\frac{\xi}{k}\right)^2 \qquad \text{[hr]}, \qquad (6)$$

In the moment of the plate casting solidifying the expression $\xi = x_p/2$ is coincident with the plate modulus $Mp = x_p/2$. Therefore it can be expressed as follows

$$M_d = k\sqrt{\tau} \qquad \text{and} \qquad \tau = \left(\frac{M_d}{k}\right)^2 \qquad (7)$$

In such a way the solidification time τ of the plate-shaped casting is bound with its modulus. The **k** coefficient is dependent on a number of physical quantities and chemical composition of the material; in a simplified formed it has an expression as follows :

$$k = \frac{2}{\sqrt{\pi}} \cdot \frac{T_{sol}\ .b_f}{[[L + (T_{liti} - T_{sol})c_k]]\rho_k} \qquad [\text{m . hr}^{-1/2}]; \qquad (8)$$

where:

T_{sol} – solidus temperature [K]

b_f – coefficient of heat diffusivity of the mould [W.s$^{1/2}$.m^{-2}. ºC^{-1}];

L – latent heat of crystallization, [J.kg^{-1}];

c_k – specific heat of metal, [J.kg^{-1}. ºC^{-1}];

ρ – mass density of the melt, [kg . m^{-3}].

For the other geometric shapes of castings the thermophysical conditions of solidification are more complicated. As mentioned the cause consists in different metal volume and cooling surface.

The M_C modulus expresses conditions for determination of relative solidification time of geometrically different castings. Dependences on Figure 2 show that the plate casting solidifies according to parabolic dependence but the cylinder and ball curves (2, 3) deflect from inflection points upwards and castings will solidify in a shorter time τ_c (0.5) and τ_s (0.33). This change of the curves direction is caused by the increased solidification rate of the internal part of castings. As it has been stated it is caused by irregular change of the melt volume V and the surface A during solidification. Though those interesting processes seem to be unreal at first scrutiny when the internal zone of the casting (without technological intervention) cools down faster than its surface, they are quite regular physical solidification process.

Another physical factor acting in the same sense as the M_C modulus and increases the solidification rate of the central part of the cylinder and sphere castings is the change of thermal regime between the casting and the mould and in the casting itself.

Heat transfer from the casting is determined by a relation (Figure 4) as follows:

$$q_1 = \alpha.\ \Delta T \qquad [\text{W . m}^{-2}]; \qquad (9)$$

where:

α – heat transfer coefficient, [W . m^{-2} .K^{-1}];

ΔT – thermal gradient between the casting and the mould, [K].

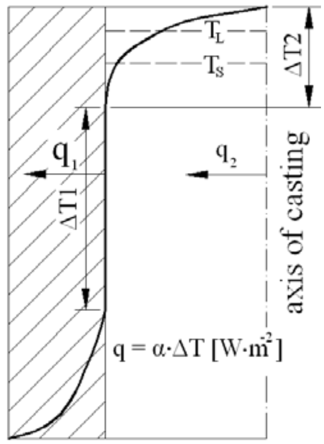

Fig. 4. Schematic drawing of thermal gradients and heat flow in a casting and a mould

At the beginning of solidification the thermal gradient ΔT_1 between the casting and the mould is a high one and the heat flow q_1 from the casting into the mould predominates (Figure 4).

With regard to thermal gradient in a casting ΔT_2 happens to heat flow from the centre of casting to its surface. Behind certain period both heat flows equalize and farther dale turns heat flow q_2.

4. Experiments and discussion of results

The verification of theoretical conclusions was carried out by the measurement of thermal fields in heavy duty steel castings in sand and metallic moulds with the aid of thermocouples Pt-PtRh 10 with graphical registration.

It can be documented by results of experimental measurements of the solidification process of a steel cylinder casting of Ø 550 mm (and an ingot 600 x 600 mm) given in Figure 5. Gradient ΔT_1 equals to 1520 ºC and the large heat flow from the casting (heat shock) induces high solidification rate (Figure 6.). Thus the casting surface cools fast, the mould is heated, the ΔT_1 gradient and the heat flow q_1 from the casting are falling down. It causes the decrease of solidification rate $\mathbf{v_{sol}}$ down to minimum value of 2.8 mm . min^{-2} in the inflection point in time of 45 min from the solidification beginning, in the casting depth of 125 mm (Figure 6.). It also decelerates the growth of the solidified layer up to the inflection point on the solidification curve in Figure 7.

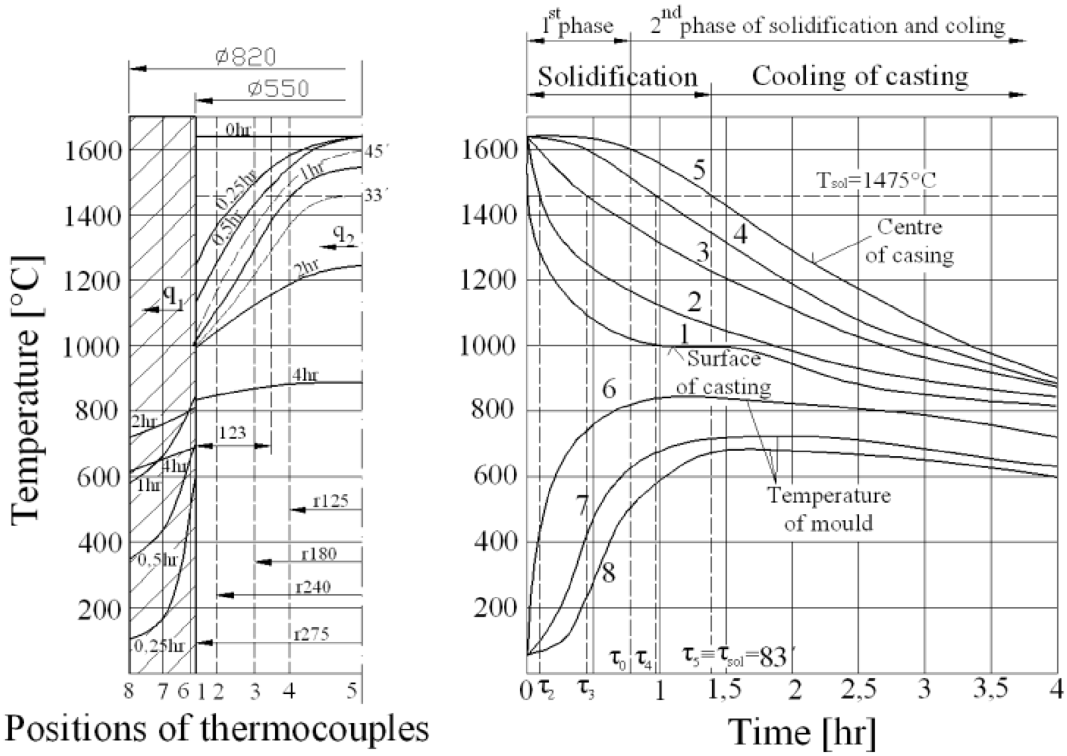

Fig. 5 a, b. Temperature field in steel cylinder casting with diametre 550 mm and in a metallic mould

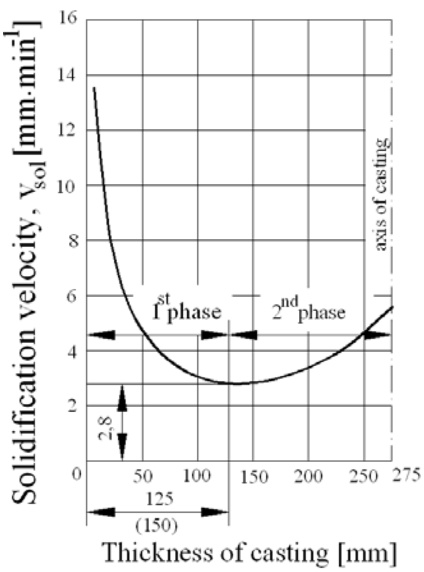

Fig. 6. Solidification velocity in cross section of steel cylinder with diameter 550 mm

Under the influence of growing thermal gradient ΔT_2 the heat flow q_2 starts to flow in the casting cross-section from the centre towards the casting surface with cooling the central zone and the solidification rate increases from the inflection point (Figure 8). At the same time the solidification curve is bent from the inflection point upwards from the parabolic dependence and solidification time shortens, it equals to 83 min. (Figure 5a).

Inflection points are determined from cooling curves. It is evident from Figure 5b that initial high rate of casting surface cooling (1) is slowing down; on the contrary to it the centre (5) with increasing the thermal gradient ΔT_2 and the heat flow q_2 is cooled faster. In certain time these cooling rates are equalized. This time is determined by time derivation or by parallel tangents to both curves) in the same time (45 min.) .

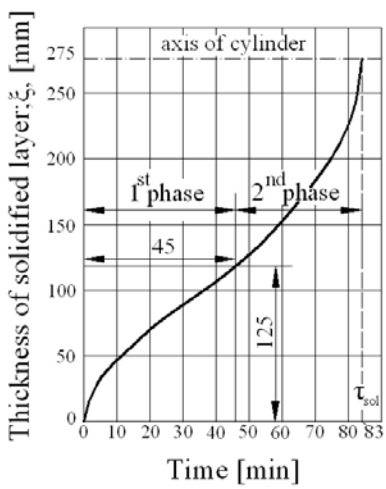

Fig. 7. The solidification curve of steel cylinder with diameter 550 mm

For comparing those results for the cylinder the data from measuring the solidification time (that equals to 109 min.) of a steel ingot of 600 x 600 mm are given here (Figure 8.). The inflection point was achieved after 52 min. in depth of 130 mm. Minimum solidification rate is 1.7 mm . min.$^{-2}$. The results are favourable, they correspond to a slight increase of the ingot cross-section in comparison with the cylinder. The inflection point position can be slightly changed according to casting temperature, the intensity of heat removal by the mould etc. A principal one is the change of the casting thermal regime that results in higher solidification rates in the centre and the decrease of cooling rates of external casting layers. Those findings are important for next processes of casting solidification and cooling. They divide the whole process of casting solidification to two phases (periods), each with different solidification process and the formation of casting defects. First phase (I^{st}) in this cylinder casting is 45 min. long with thickness of the solidified layer of 125 mm. The solidification process is managed in this phase by the surface layers. Due to their fast solidification and shrinkage a stress state is developed in the casting. In external layers there are tensile stresses, in the central casting part there are compressive stresses. Therefore in first phase there are conditions for casting deformation or for formation of surface tears. Their characteristic course is traced by the grain boundary (they are formed under high temperatures).

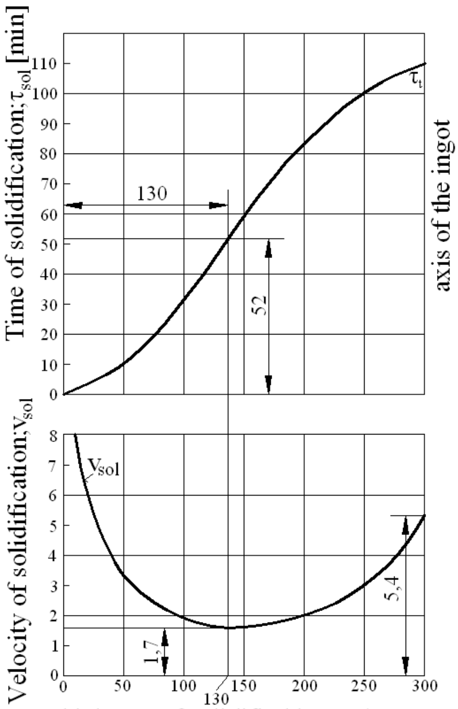

Fig. 8. The solidification velocity in cross section of stell ingot 60 0x 600 mm, weight 4,8 t

In second solidification phase it is in turn. Processes of casting solidification and cooling down to room temperatures are managed by the thermal regime of the casting centre. Second phase starts with high thermal gradient ΔT_2 = 420 ºC in casting cross-section (Figure 4) and thus with heat flow q_2 from the centre towards the surface. In such a way the central zone is highly cooled and solidification rate increases from the inflection point up to the casting axis (Figure 7). At the same time the solidification curve deflects behind the inflection point in Figure 8 and casting solidification time shortens. In second phase besides the heat flow q_2 the M_C modulus of the casting also participates in a less share in increasing the solidification rate in the central casting part. Total rate increase for the cylinder (Figure 6) and ingot (Figure 8) is about 3 mm . min^{-2}.

It is necessary to remind that this regular process acts unfavourably on internal casting quality. Increased rate of central part solidification shortens its solidification time and in such a way it limits the time of the physical process course of the melt capillary filtration (penetration) through the dendrite network in the solidification zone into the solid region what increases the shrinkage porosity volume in the casting. On the contrary the melt penetration with low overheating requires a longer time for its full development and the reduction of shrinkage porosity content. This conclusion has been experimentally proved [17].

Thermal processes in second solidification phase with intensive cooling (curve 5 on Figure 5.) are of high importance also for the reason of the stress condition in the casting during its cooling and thermal treatment. They totally change the stress nature as in second phase in the central zone there are tensile stresses while in external layers there are compressive ones. During cooling it causes internal cracks (under high temperatures) or cold cracks (under low temperatures in the elastic zone of the material). Constant stresses are given by a relation as follows:

$$\sigma = E \,.\, \alpha.\, \Delta T \qquad [\mathrm{MPa}]; \qquad (10)$$

where:
E – Young's modulus, [MPa]
α – thermal shrinkage coefficient, [1/K]
ΔT – thermal gradient in the casting, [K]

A factor influencing the stress condition is the thermal gradient ΔT. There is an endeavour to reduce it by controlled cooling. Its lower value is achieved with less difference of the casting wall thickness, with casting the moulds with lower cooling effect, and with lower cooling rate. Always it is necessary to consider that the highest permanent stress in the casting is after having it fully cooled down. Therefore besides the other thermal regime the thermal regime for decreasing the stress must be included for steel castings too.

5. Conclusions

The casting modulus expressing the relation of the casting volume to its surface and discovered for solidification conditions by N. Chvorinov means a considerable contribution for manufacturing technology and for study of solidification process. It defines the accurate solidification conditions of basic bodies – geometric configurations – for the casting design. It has that special feature that in the casting formation process it is not a constant but during solidification process it falls down from initial value as the rest melt volume decreases faster than its cooling surface. Thus it causes the solidification acceleration. Therefore in first solidification phase, when under the influence of falling heat transfer from the casting the solidification rate also decreases, it acts contrariwise and it endeavours to limit this fast decrease with its less influence. In second solidification phase, when under the influence of the thermal regime the casting cross-section central zones cool and solidify faster, the modulus effect is in agreement with this process and therefore it increases the cooling intensity and solidification rate. The above achieved results enlarged theoretical calculations of Chvorinov [9,13] , Jamar [18] and recent research of Santos and Garcia [19].

Having evaluated this regular natural course of the solidification process, when in second solidification phase the solidification rate increases in the casting centre, it is unsuitable and even harmful from the point of view of casting quality. Increased solidification rate shortens the total solidification time what limits the course of the physical process of filtration (penetrating) the melt through the dendrite network in the solidification zone into the solidus region what increases the shrinkage porosity volume in the central part of the casting cross-section. On the contrary the penetration requires a longer solidification time for its full development. Similar results have been obtained for a plate casting cast in a bentonite mould under temperature of 1100 ºC and a casting in a mould without heating where the breaking force was substantially lower.

Second disadvantage of accelerated solidification of the casting centre consists in the change of the casting stress condition. Central regions are stressed with tensile stress causing the shape deformations or internal cracks, in cylinders e.g. even over the all cross-section.

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