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Identification of the Heat Transfer Coefficient in the Inverse Stefan Problem by Using the ABC Algorithm

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Abstract

A procedure based on the Artificial Bee Colony algorithm for solving the two-phase axisymmetric one-dimensional inverse Stefan problem with the third kind boundary condition is presented in this paper. Solving of the considered problem consists in reconstruction of the function describing the heat transfer coefficient appearing in boundary condition of the third kind in such a way that the reconstructed values of temperature would be as closed as possible to the measurements of temperature given in selected points of the solid. A crucial part of the solution method consists in minimizing some functional which will be executed with the aid of one of the swarm intelligence algorithms - the ABC algorithm.

Keywords: Solidification process, Application of information technology to the foundry industry, Two-phase inverse Stefan problem, Swarm intelligence, ABC algorithm

1. Introduction

In this paper we present an algorithm for solving the two-phase axisymmetric one-dimensional inverse Stefan problem with the third kind boundary conditions and the temperature measurements taken in selected points of the solid phase as an additional information. Solving of the considered problem consists in reconstructing the form of heat transfer coefficient appearing in boundary condition of the third kind, so that the temperature in the given points of solid phase would have the closest values as possible to the known control values.

Swarm Intelligence is a group of algorithms representing a part of the artificial intelligence in which a common general solution of the problem is constructed on the basis of particular solutions determined independently by members of the swarm. The expression was formulated by Gerardo Beni and Jing Wang in 1989 in the context of cellular robotic systems [1] and it denotes the collective behavior of decentralized, self-organized, natural or artificial individuals. A group of such individuals can be, for example, the swarms of insects, behavior of which became an inspiration for inventing such algorithms like the Artificial Bee Colony algorithm (ABC) or Ant Colony Optimization algorithm (ACO).

The Artificial Bee Colony algorithm imitates the technique of searching for the nectar around the hive by colony of bees. After localizing some good source of food, the bee-scout collects a sample of the nectar and returns to the hive for informing the other bees, with the aid of a special waggle dance, about the position of the available source. After the dance, the bees-viewers leave the hive and fly in the direction of the discovered source of nectar, whereas the bee-scout can stay in the hive or leave it for searching a new source of food or for exploring the discovered one. More detailed information about the natural inspiration of the ABC algorithm can be found in [2,3] and examples of its application for solving the inverse heat conduction problems, made by the authors, are presented in [4-6]. In general, application of the modern and popular in recent time algorithms of artificial intelligence, like the Ant Colony Optimization algorithm [7] or the evolutionary algorithm [8], for solving the inverse heat conduction problems becomes more and more frequent, because these new approaches ensure very good results, on a par with more classical methods [9-11].

In the current paper we consider the inverse Stefan problem which is a mathematical problem serving for description of thermal processes with the phase transitions, like, for example, solidification of pure metals, melting of ice, freezing of water, in which, however, some of the input information is missing and must be reconstructed. The sought element can be, for instance, the form of initial condition, boundary conditions or parameters of material, whereas the additional information, compensating the missing one, can be the measurements of temperature, location of the freezing front or its velocity. Inverse Stefan problem belongs to the group of ill posed problem which means that its solution may not exist or may be neither unique nor stable. Therefore the constant efforts are made to propose some efficient procedures of determining the approximate solution of inverse Stefan problem. For instance, in paper [12] the inverse Stefan problem is solved by using the Lie-group shooting method. Authors of paper [13] have presented the possibility of application of the optimal control for solving the inverse design Stefan problem concerning the temperature selection on the boundary. Application of the heat balance integral method for solving the inverse design Stefan problem is showed in paper [14]. In papers [15,16] the problem of determination of the interface location is formulated as a geometrical inverse problem. Summary bibliography review for the methods of solving the inverse Stefan problem is presented in monograph [17]. In spite of all mentioned approaches, the literature devoted to the inverse Stefan problem is much more poor in comparison with literature concerning the direct problems (for example [18-21]).

2. Artificial Bee Colony algorithm

ABC algorithm imitates the technique of communication between bees: after discovering the attractive source of nectar the bee (called the scout) flies back with the sample of nectar to the hive for informing the other bees (called the viewers) about the position of sources of nectar, with the aid of a special kind of dance called the waggle dance. In details, the ABC algorithm can be listed in form of the following steps.

<u>Initialization of the algorithm</u>

1. Initial data:
   - SN - (Swarm Number) number of the bees-scouts (= number of the bees-viewers);
   - D - dimension of the explored sources x^i, $i=1,...,SN$;
   - $\lim$ - number of the corrective flights around sorce x^i;
   - MCN - maximal number of cycles.
2. Initial population - random selection of the initial sources localizations represented by the D- dimensional vectors x^i, $i=1,...,SN$;
3. Calculation of the values $F(x^i)$, $i=1,...,SN$, for the initial population, where F denotes the minimized function.

<u>Main algorithm</u>

1. Modification of the sources localizations by the bees-scouts.

a) Every bee-scout modifies the position x^i according to the formula:

$$v_j^i = x_j^i + \phi_{ij}(x_j^i - x_j^k), \qquad j \in \{1,...,D\},$$

where: $\left.\begin{array}{l} k \in \{1,...,SN\},\ k \neq i, \\ \phi \in [-1,1] \end{array}\right\}$ - randomly selected numbers.

b) If $F(v^i) \leq F(x^i)$, then position v^i replaces x^i. Otherwise, position x^i stays unchanged.

Steps a) and b) are repeated $\lim$ times. We take: $\lim = SN \cdot D$.

2. Calculation of the probabilities P_i for the positions x^i selected in step 1. We use the formula:

$$P_i = \frac{fit_i}{\sum_{j=1}^{SN} fit_j}, \qquad i=1,...,SN,$$

where:
$$fit_i = \begin{cases} \dfrac{1}{1+F(x^i)}, & if \quad F(x^i) \geq 0, \\ 1+|F(x^i)|, & if \quad F(x^i) < 0. \end{cases}$$

3. Every bee-viewer chooses one of the sources x^i, $i=1,...,SN$, with probability P_i. Of course, one source can be chosen by a group of bees.
4. Every bee-viewer explores the chosen source and modifies its position according to the procedure described in step 1.
5. Selection of x^{best} for the current cycle - the best source among the sources determined by the bees-viewers. If the current x^{best} is better that the one from the previous cycle, it is accepted as x^{best} for the entire algorithm.
6. If in step 1, the bee-scout did not improve the position x^i (x^i did not change), it leaves source x^i and moves to the new one, according to the formula:

$$x_j^i = x_k^{\min} + \phi_{ij}(x_k^{\max} - x_k^{\min}), \qquad j=1,...,D,$$

where $\phi_{ij} \in [0,1]$.

Steps 1-6 are repeated MCN times.

3. Formulation of considered problem

Let the boundary of region $\Omega = [0,b] \times [0,t^*] \subset R^2$ be divided into following parts (see Figure 1):

$$\Gamma_0 = \{(x,0) : x \in [0,b]\},$$
$$\Gamma_{11} = \{(0,t) : t \in [0,t_k)\},$$
$$\Gamma_{12} = \{(0,t) : t \in [t_k,t^*]\},$$
$$\Gamma_{21} = \{(b,t) : t \in [0,t_p)\},$$
$$\Gamma_{22} = \{(b,t) : t \in [t_p,t^*]\},$$

where the initial condition and boundary conditions are defined. Symbol Ω_1 denotes the subregion of Ω taken by the liquid phase, whereas Ω_2 describes the subregion taken by the solid phase. The interface is denoted by Γ_g, location of which is described by function $x = \xi(t)$.

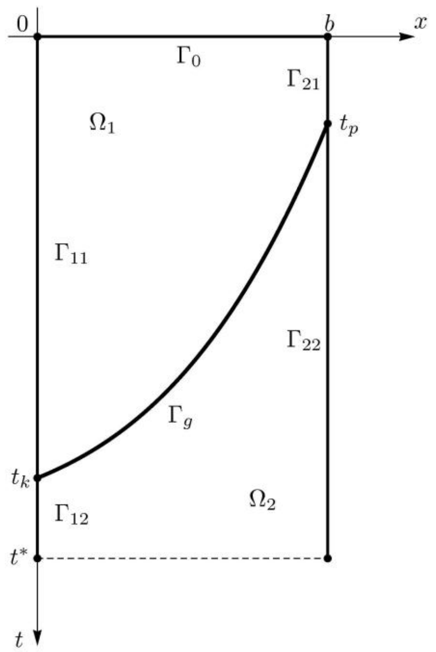

Fig. 1. Domain of the problem

In selected points of the solid phase ($(x_i,t_j) \in \Omega_2$) the values of temperature are known:

$$T_2(x_i,t_j) = U_{ij}, \quad i = 1,2,..., N_1, \quad j = 1,2,..., N_2, \tag{1}$$

where N_1 denotes the number of sensors (thermocouples) and N_2 means the number of measurements taken from each sensor.

The problem consists in determining the function $\alpha(t)$ defined on boundaries Γ_{2k} (for $k = 1,2$) such that the function $\xi(t)$ describing the interface position and the distributions of temperature T_k in regions Ω_k ($k = 1,2$), calculated for reconstructed $\alpha(t)$, would satisfy the axisymmetric one-dimensional heat conduction equation inside the regions Ω_k (for $k = 1,2$):

$$c_k \rho_k \frac{\partial T_k}{\partial t}(x,t) = \frac{1}{x} \frac{\partial}{\partial x}\left(\lambda_k x \frac{\partial T_k}{\partial x}(x,t) \right), \tag{2}$$

the initial condition on boundary Γ_0 ($T_0 > T^*$):

$$T_1(x,0) = T_0, \tag{3}$$

the homogeneous boundary conditions of the second kind on boundaries Γ_{1k} ($k = 1,2$):

$$\frac{\partial T_k}{\partial x}(0,t) = 0, \tag{4}$$

the boundary conditions of the third kind on boundaries Γ_{2k} ($k = 1,2$):

$$-\lambda_k \frac{\partial T_k}{\partial x}(b,t) = \alpha(t)(T_k(b,t) - T_\infty), \tag{5}$$

as well as the condition of temperature continuity and the Stefan condition on interface Γ_g:

$$T_1(\xi(t),t) = T_2(\xi(t),t) = T^*, \tag{6}$$

$$L \rho_2 \frac{d\xi}{dt} = -\lambda_1 \frac{\partial T_1(x,t)}{\partial x}\bigg|_{x=\xi(t)} + \lambda_2 \frac{\partial T_2(x,t)}{\partial x}\bigg|_{x=\xi(t)}, \tag{7}$$

where c_k, ρ_k and λ_k are, respectively, the specific heat, mass density and thermal conductivity in liquid phase ($k = 1$) and solid phase ($k = 2$), $\alpha(t)$ denotes the heat transfer coefficient, T_0 - the initial temperature, T_∞ - the ambient temperature, T^* - the solidification temperature, L describes the latent heat of fusion and, finally, t and x refer to the time and spatial location.

Direct Stefan problem described by equations (2)-(7) for the fixed form of heat transfer coefficient is solved by using the alternating phase truncation method [22], in result of which, among others, the course of temperature in solid phase can be determined. Values of temperature U_{ij}, calculated in this way, are considered as the exact values and are treated as the benchmark for comparing the results of solving the inverse problem. For examining the influence of input data errors into the exactness of results and stability of the algorithm, the numerical experiment is executed. In this experiment the exact values of temperatures U_{ij} are burdened by the random errors which simulate the measurements values and represent the input data.

Method of solving the inverse Stefan problem consists exactly in finding the solution of direct Stefan problem, defined by equations (2)-(7), for the fixed form of heat transfer coefficient α, which enables to find the values of temperature $T_{ij} = T_2(x_i, t_j)$ associated with the respective α. By using the calculated temperatures T_{ij} and given temperatures U_{ij} the following functional is constructed:

$$J(\alpha) = \sqrt{\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}(T_{ij} - U_{ij})^2} \tag{8}$$

representing the error of approximate solution, by minimizing of which the values of parameter α assuring the best approximation of temperature will be found. For minimizing functional (8) the ABC algorithm is used.

4. Numerical example

Proposed approach will be tested by executing the experiment in which the solved problem is modelled by means of the two-phase axisymmetric one-dimensional Stefan problem described by equations (2)-(7) for the following values of parameters: $b = 0.08$ [m], $\lambda_1 = 104$ [W/(m·K)], $\lambda_2 = 240$ [W/(m·K)], $c_1 = 1290$ [J/(kg·K)], $c_2 = 1000$ [J/(kg·K)], $\rho_1 = 2380$ [kg/m^3], $\rho_2 = 2679$ [kg/m^3], $L = 390000$ [J/kg], solidification temperature $T^* = 930$ [K], ambient temperature $T_\infty = 298$ [K] and initial temperature $T_0 = 1013$ [K]. The exact form of the sought heat transfer coefficient α [W/(m^2·K)] for the considered process is known and is equal to:

$$\alpha(t) = \begin{cases} 1200 & for \quad t \in [0,90], \\ 800 & for \quad t \in (90,250], \\ 250 & for \quad t \in (250,1000]. \end{cases}$$

Such assumption about the heat transfer coefficient changing at two predetermined times is often used in modeling the boundary conditions for the continuous casting processes.

For solving the direct Stefan problem associated with the investigated inverse problem we use the finite difference method for the mesh of steps equal to $\Delta t = 0.1$ and $\Delta x = b/500$ with applying the alternating phase truncation method. Whereas, for constructing functional (8) we use the exact values of temperature and values noised by the random error of 1%, 2% and 5%. The thermocouple is located in point $x = 0.07\,[m]$.

The ABC algorithm is executed for 8 bees ($SN = 8$), maximal number of cycles equal to 10 ($MCN = 10$) and the initial population of individuals is randomly selected from the range $[0,500]$. Because of the heuristic nature of ABC algorithm, which means that every running of this algorithm can give slightly different results, the calculations are evaluated for 30 times and the approximate values of reconstructed parameter are received by averaging the obtained results.

In Figures 2 and 3 the relative errors of reconstructed values of coefficients α_i $(i = 1,2,3)$ are presented, in dependence on the number of cycles, calculated for the measurements perturbated by the error of 2% and 5%, respectively.

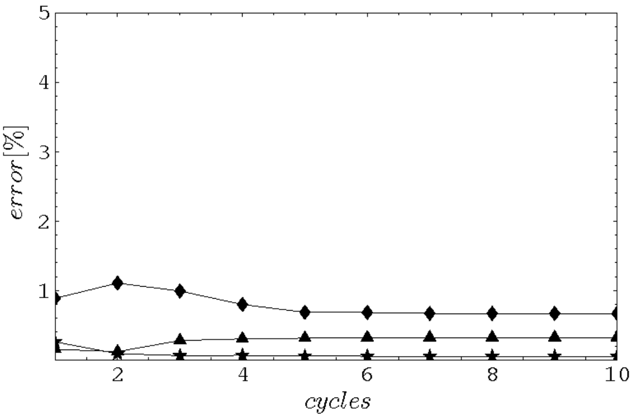

Fig. 2. Relative error of coefficients α_i $(i = 1,2,3)$ reconstruction for the successive iterations (∗ for α_1, ▲ for α_2, ♦ for α_3) for input data burdened by 2% error

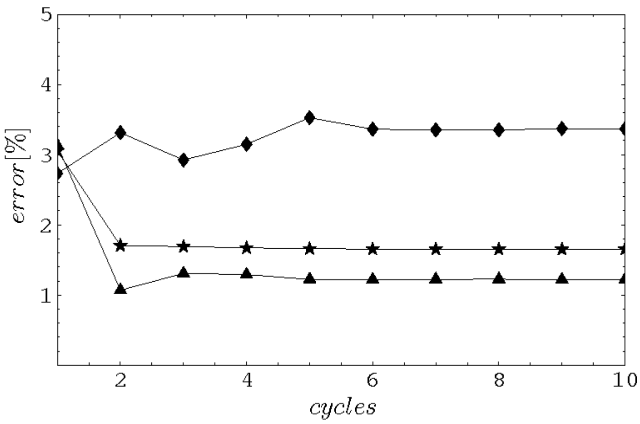

Fig. 3. Relative error of coefficients α_i $(i = 1,2,3)$ reconstruction for the successive iterations (∗ for α_1, ▲ for α_2, ♦ for α_3) for input data burdened by 5% error

The figures show that in both cases the reconstruction error is smaller than the input data and satysfying results are obtained for only 3 cycles. Further increase of the cycles number does not improve the results any longer. Comparisons of the temperature values given and reconstructed in control point $x = 0.07$, calculated for measurements error of 2% and 5%, are showed in Figures 4 and 5. It can be seen that in both cases the exact and approximate courses of temperature cover.

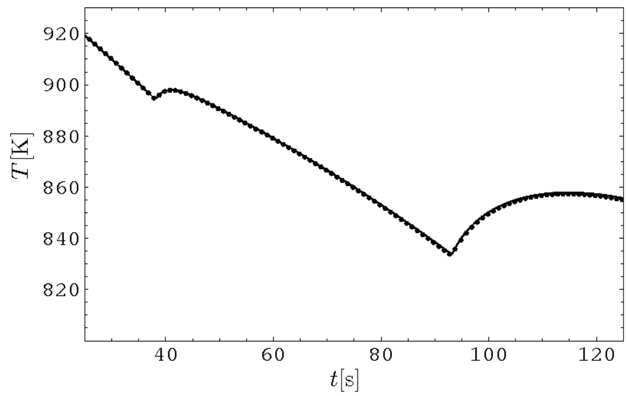

Fig. 4. Comparison of the given (solid line) and reconstructed (dashed line) values of temperature in control point $x = 0.07$ for input data burdened by 2% error

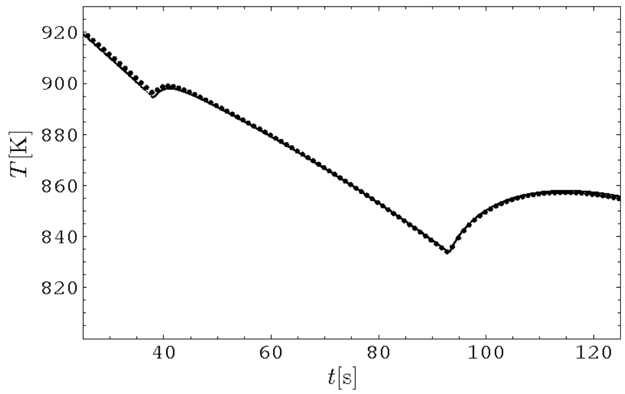

Fig. 5. Comparison of the given (solid line) and reconstructed (dashed line) values of temperature in control point $x = 0.07$ for input data burdened by 5% error

Statistical elaboration of results obtained in 30 executions of the procedure, for 5 and 10 cycles and for various noises of input data error, are compiled in Tables 1 and 2. It can be observed that in each case the reconstruction errors, for the identified heat transfer coefficient as well as for the approximate temperature, are much smaller than input data errors. One can even notice that results for 2% input data error in α_1 reconstruction are better than the results for 1% error, but one should undestand that it has happend only for this specific selection of initial values of the sought parameters. Important is that statistical data show the repeatability of obtained results and small values of standard deviations and coefficients of variation confirm stability of the procedure.

Table 1.
Mean values ($\overline{\alpha_i}$), absolute (Δ_{α_i}) and relative (δ_{α_i}) errors of the reconstructed heat transfer coefficients together with the absolute (Δ_T) and relative (δ_T) errors of temperature reconstruction obtained for various noises of input data in 5 and 10 cycles of the procedure

noise	*MCN*	*i*	$\overline{\alpha_i}$	Δ_{α_i}	δ_{α_i}[%]	Δ_T	δ_T[%]
1%	5	1	1205.699	5.699	0.475	4.757	0.570
		2	798.849	1.151	0.144		
		3	250.508	0.508	0.203		
	10	1	1205.681	5.681	0.473	4.755	0.570
		2	798.856	1.144	0.143		
		3	250.488	0.488	0.195		
2%	5	1	1200.643	0.643	0.054	1.466	0.176
		2	802.505	2.505	0.313		
		3	251.713	1.713	0.685		
	10	1	1200.626	0.626	0.052	1.443	0.173
		2	802.582	2.582	0.323		
		3	251.672	1.672	0.669		
5%	5	1	1180.067	19.933	1.661	9.996	1.199
		2	809.777	9.777	1.222		
		3	258.815	8.815	3.526		
	10	1	1180.168	19.832	1.653	9.996	1.199
		2	809.763	9.763	1.220		
		3	258.418	8.418	3.367		

Table 2.
Standard deviations (S_i), coefficients of variation (V_i) and maximal relative errors (r_i) of the reconstructed heat transfer coefficients obtained for various noises of input data in 5 and 10 cycles of the procedure

noise	*MCN*	*i*	S_i	V_i[%]	r_i[%]
1%	5	1	2.030	0.168	1.138
		2	1.283	0.161	0.783
		3	1.029	0.411	1.065
	10	1	2.032	0.169	1.138
		2	1.295	0.162	0.783
		3	1.021	0.408	1.065
2%	5	1	1.401	0.117	0.341
		2	1.246	0.155	0.595
		3	1.873	0.744	1.832
	10	1	1.404	0.117	0.341
		2	1.174	0.146	0.595
		3	1.853	0.736	1.832
5%	5	1	9.626	0.816	5.062
		2	5.301	0.655	4.019
		3	1.385	0.535	5.043
	10	1	9.643	0.817	5.062
		2	5.271	0.651	4.005
		3	0.994	0.384	5.037

4. Conclusions

The paper contains a proposal of the method for solving the two-phase axisymmetric one-dimensional inverse Stefan problem with boundary condition of the third kind. Important part of this procedure is the minimization of appropriate functional realized with the aid of ABC algorithm. Presented results indicate that the proposed approach ensures the approximate solution rapidly convergent to exact solution and perturbated by the error much smaller than the error of input data for relatively small number of cycles. That confirms the usefulness of bees algorithm in investigating the thermodynamic processes. Moreover, satisfying approximations were determined very quickly and the successive cycles did not improve significantly the results. That is why the proposed procedure can be used for determining the starting point for the more traditional numerical method of solving such kind of problems, which requires a good starting point and maybe will give the possibility to improve the final results.

The advantages of using approach with the ABC algorithm are at least two - the time needed for finding the global solution is respectively short and the only assumption needed by the algorithm is the existence of solution. If the solution of optimized problem exists, it will be found with some given precision of course and the time of its receiving depends only on the time of solving the associated direct problem.

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