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Determination of Reliability Index and Weibull Modulus as a Measure of Hypereutectic Silumins Survival

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Abstract

The first part of the study describes the methods used to determine Weibull modulus and the related reliability index of hypereutectic silumins containing about 17% Si, assigned for manufacture of high-duty castings to be used in automotive applications and aviation. The second part of the study discusses the importance of chemical composition, including the additions of 3% Cu, 1,5% Ni and 1,5% Mg, while in the third part attention was focussed on the effect of process history, including mould type (sand or metal) as well as the inoculation process and heat treatment (solutioning and ageing) applied to the cast AlSi17Cu3Mg1,5Ni1,5 alloy, on the run of Weibull distribution function and reliability index calculated for the tensile strength R_m of the investigated alloys.

Keywords: Casting quality management, Reliability, Weibull modulus, Hypereutectic silumins, Mechanical properties

1. Introduction

Cast alloys used for high-duty parts operating in automotive industry and aviation should, besides excellent mechanical properties, offer also stable and failure-free performance. A good measure of the reliability of a selected examined property, e.g. the tensile strength Rm, is estimation of the value of Weibull modulus, closely related with the, so called, part's survival probability [1].

The effect of chemical composition and process history applied in manufacture of the Al-Si-Me alloys with Cu, Ni and Mg was traced using Weibull distribution function, the values of this distribution, and the values of the, so called, reliability index. The analysis of the technological process history included type of foundry mould and processes of modification and heat treatment. An additional argument for undertaking the investigation was the lack (so far) of a domestic counterpart of these alloys, widely used in West European countries and USA for, among others, cast pistons and cylinder heads operating in I.C. engines. Using the results of own investigations carried out on a wide scale [2], it has been concluded that a counterpart of the above mentioned alloys can be the silumin containing about 17% Si, with additions of 3% Cu, 1,5% Ni and 1,5% Mg. In this context, of particular significance are the methods determining the admissible risk of failure (the survival probability) of parts put in service. In modern techniques of testing parts for their mechanical resistance, it is the statistical analysis that plays the role of a tool effectively aiding the tests and determining the range of application of structural materials for operation under high-duty conditions. Quite important is also careful analysis of the individual stages of alloy fabrication.

2. Methods of investigation

In statistical analysis, the calculation sheet of Excel v. 2000 made by Microsoft and programs: Statistica v. 7.1 PL offered by StatSoft and MedCalc v. 9.1.0.1 were used.

Detailed information on the measurement of the tensile strength R_m of a modified Al-Si-Cu-Mg-Ni alloy cast in sand moulds is given in Figure 1.

Rm [MPa] Al-Si-Cu-Mg-Ni alloy			
Specimen no.	Sand mould + modification	Specimen no.	Sand mould + modification
1	187	27	201
2	191	28	195
3	189	29	201
4	193	30	203
5	191	31	193
6	189	32	197
7	189	33	199
8	190	34	196
9	195	35	196
10	194	36	199
11	191	37	198
12	195	38	199
13	193	39	199
14	196	40	199
15	190	41	199
16	191	42	199
17	195	43	198
18	197	44	194
19	194	45	195
20	192	46	202
21	193	47	195
22	192	48	196
23	196	49	198
24	197	50	199
25	196	51	200
26	197	52	197

Fig.1. The results of the tensile strength R_m measurement taken on a modified Al-Si-Cu-Mg-Ni alloy cast in sand moulds

For the first stage of statistical analysis, the parameters of descriptive statistics shown in Figure 2 were selected.

PARAMETER	*VALUE*
N	52
Mean	195,38
Confidence -95%	194,349
Confidence +95%	196,421
Median	196
Minimum	187
Maximum	203
Quartile 25%	193
Quartile 75%	198,5
Range	16
Quartile range	5,5
Variance	13,849
Standard deviation	3,721
Standard Error of Main	0,516
Skewness	-0,207
Kurtosis	-0,568

Fig.2. Basic parameters of descriptive statistics used for the data from Figure 1

The determination of some parameters shown in Figure 2 (e.g. confidence interval for the mean) makes sense only when a close-to-normal distribution of the variable is obtained [3]. To know this, it is necessary to check the null hypothesis H_0, assuming that the distribution of the examined variable is consistent with the normal one, which can be done using, e.g. Kolmogorov-Smirnov consistency test or Shapiro-Wilk test.

The confidence interval calculated for the results of R_m from the lower limit equal to about 194,35 to 196,42 [MPa] is an important factor showing the alloy reliability in respect of this parameter, since the interval covers the true mean value of R_m with the assumed probability equal to 0,95.

The determined values of the lower quartile (also known as 25% quartile) and upper quartile (also known as 75% quartile) belong to the positional measures of location, while the respective values denote that 25% R_m values are below 193 [MPa] and, at the same time, 75% R_m are above 198,5 [MPa]. So, a half (50% - 26 results) of the obtained R_m values are comprised within the range of 5,5 [MPa], which forms the, so called, quartile range (Fig.1).

At the second stage of the statistical analysis of the life of the examined alloy, the obtained values of R_m were divided into 13 classes. For this purpose, having arranged the values in an increasing order, the size of the class interval w_p was determined using the following equation:

$$w_p = \frac{\max(R_m) - \min(R_m)}{12} \tag{1}$$

For the examined alloy this value amounted to about 1,33 [MPa]. Next, for each of the obtained values of R_m a cumulative percentage of samples that exceeded the adopted threshold limit was determined using the following equation:

$$\% \, próbek = \frac{52 - n_s}{52} \cdot 100\% \tag{2}$$

where: n_s – is the cumulative size of sample in which the value of the examined property has exceeded the adopted threshold limit. Then, the *sigma0* value for which 37% of the samples have exceeded this value in respect of R_m was determined (Fig. 3)

After determination of the upper interval limits, where as an upper limit of the first interval the value $\min R_m$, was adopted and each of the remaining intervals was open from the lower limit and closed from the upper one, the number of samples (the, so called, frequency) belonging to the described intervals n_k was determined, using Excel table function *CZĘSTOŚĆ*() Y - FREQUENCY (Fig. 3).

	Al-Si-Cu-Mg-Ni alloy									
Interval size (w_p)	Sigma	Frequency (n_k)	Cumulative (n_{ks})	52-n_{ks}	Pi	1/Pi	ln(1/p)	ln(ln(1/p))	Sigma/Sigma0	ln(Sigma/Sigma0)
1,33	187,00	1	1	51	0,9808	1,020	0,019	-3,9416	0,9492	-0,0521
Sigma0	188,33	0	1	51	0,9808	1,020	0,019	-3,9416	0,9560	-0,0450
197	189,67	3	4	48	0,9231	1,083	0,080	-2,5252	0,9628	-0,0379
	191,00	6	10	42	0,8077	1,238	0,214	-1,5438	0,9695	-0,0309
	192,33	2	12	40	0,7692	1,300	0,262	-1,3380	0,9763	-0,0240
	193,67	4	16	36	0,6923	1,444	0,368	-1,0004	0,9831	-0,0171
	195,00	9	25	27	0,5192	1,926	0,655	-0,4225	0,9898	-0,0102
	196,33	6	31	21	0,4038	2,476	0,907	-0,0979	0,9966	-0,0034
	197,67	5	36	16	0,3077	3,250	1,179	0,1644	1,0034	0,0034
	199,00	11	47	5	0,0962	10,400	2,342	0,8509	1,0102	0,0101
	200,33	1	48	4	0,0769	13,000	2,565	0,9419	1,0169	0,0168
	201,67	2	50	2	0,0385	26,000	3,258	1,1811	1,0237	0,0234
	203,00	2	52							
	Razem	52	=I4+J3	=52-J4	=1/L3	=LN(M3)		y variable		x variable
	=CZĘSTOŚĆ(E3:E54;H3:H15)				=K3/$I$16		=LN(N3)		=H3/$G$5	=H3/$G$5

Fig.3. Determination of values necessary for the estimation of Weibull modulus

For the estimation of Weibull modulus m the authors used the function allowing for a relationship between the "survival" probability p (i.e. the cumulative probability that the examined property will exceed the adopted threshold limit, counted from its maximum value), the selected property of material (e.g. σ (sigma) – that is R_m) and modulus m, given by M. Ashby and D. Jons [2]:

$$p = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{3}$$

where: *sigma0* (σ_0) is the value for which 37% of samples exceed this value in respect of the examined property.

Function (3) was reduced to its linear form by double two-sided logarithmic operation done with a natural logarithm. As a result of this operation, a function model in the form $y = a \cdot x$: was obtained (Fig.3)

$$\ln\left[\ln\left(\frac{1}{p}\right)\right] = m \ln \frac{\sigma}{\sigma_0} \tag{4}$$

Using the obtained empirical pairs of values (x_i; y_i) (Fig.3), parameters of the model of linear function of the type $y= a·x$: were estimated. For this purpose an Excel *Narzędzie - Tool* from the packet of tools *Analiza Danych - Data Analysis* called *Regresja - Regression* was used [4]. To estimate the coefficients of the function of regression, the Tool uses a method of optimisation based on minimalisation of the sum of least squares (SLS) of the deviations of empirical points from a model curve. The results of the estimation of the coefficient *a* of the function of regression, which at the same time is the searched value of Weibull modulus for the examined cast alloy, are shown in Fig. 4.

Analysis of regression and correlation				
Linear model *y=ax*: (ln(ln(1/*p*))=ln(Sigma/Sigma0)**m*)				
Regression statistics				
Multiple	0,9756			
R square	0,9517			
Sdjusted R square	0,8608			
Standard error	0,3890			
Observations	12			
	Coefficients	*Standard error*	*t Stat*	*p*
Moduł Weibulla (*m*):	*69,8362*	4,08756	17,085	0,0000
	Lower 95%	*Upper 95%*		
	60,83958	78,83288		

Fig.4. Determination of Weibull modulus value (*m*) for the examined alloy (*sigma0*=197)

The obtained model of the searched function of regression for the examined alloy is as follows:

$$\ln\left[\ln\left(\frac{1}{p}\right)\right]=69,84\ln\frac{\sigma}{197} \qquad (5)$$

A graphic representation of the model obtained in double logarithmic system is shown in Figure 9.

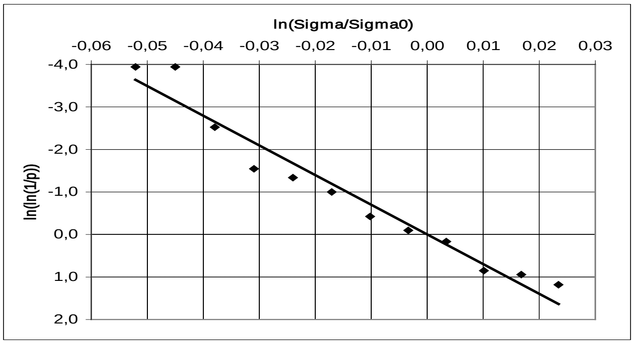

Fig.5. Graphic representation of a model of the survival function plotted in double logarithmic system

Table 1 shows the effect of chemical composition of the examined cast Al-Si17 alloys after modification and casting into metal mould on the values of Weibull modulus *m* determined for the tensile strength.

The results of the investigations indicated that the highest value of Weibull modulus *m* had Al-Si-Cu-Mg-Ni alloys. Therefore these alloys were subjected to further examinations to estimate the effect of process history on their properties.

Table 1.
Effect of the chemical composition of cast Al-Si17 alloys after modification and casting into metal mould on the values of Weibull distribution *m* determined for R_m

Rm	Metal mould + modification				
[MPa]	AlSi	AlSiCu	AlSiCuNi	AlSiCuMg	AlSiCuMgNi
Mean	147,79	165,98	166,69	175,21	215,92
Weibull modulus	**42,86**	**74,75**	**67,48**	**71,54**	**82,58**
Sigma0	149	167	168	176	217,8
SD	4,031	3,777	2,832	3,103	3,044
Variability index	**0,0273**	**0,0228**	**0,0170**	**0,0177**	**0,0141**
Wsk 1	**10,63**	**19,79**	**23,83**	**23,06**	**27,13**
Wsk 2	**15,714**	**32,849**	**39,718**	**40,395**	**58,576**
Assumed	*Wsk* 3	*Wsk* 3	*Wsk* 3	*Wsk* 3	*Wsk* 3
200	**-5,55**	**-6,73**	**-7,94**	**-5,72**	**4,32**

To interrelate the estimated values of Weibull modulus with very important (for the structural materials) values of the statistical descriptive characteristics belonging to a group of the variability measures, several indeces were proposed interrelating the estimated value *m* with: the value of standard deviation *s* – *Wsk1*, the value of variability index V_z – *Wsk2* and the assumed value of R_m and V_z – *Wsk3*:

$$Wsk1=\frac{m}{s} \quad Wsk2=\frac{m}{V_z}\cdot\frac{1}{100} \quad Wsk3=\frac{m}{100}\cdot\frac{\bar{x}-W_z}{s} \qquad (6,7,8)$$

Each of the indices should have the highest value possible and as such can serve as a criterion in the choice of material best matching the assumed operating conditions. Table 2 shows the effect of process history during manufacture of the examined cast alloys on the value of Weibull modulus *m* and on the values of the indices: *Wsk1*, *Wsk2* and *Wsk3* enabling the choice of best material for the specific operating conditions. It has been observed that the highest values of Weibull modulus and of the calculated indices had the samples of the modified Al-Si-Cu-Mg-Ni alloy cast into metal mould and subjected to heat treatment, which decided this alloy was selected for further investigations of the thermal fatigue behaviour and was proposed as a material for industrial applications.

Table 2.
Effect of process history during manufacture of cast Al-Si-Cu-Mg-Ni alloys on the values of Weibull modulus *m* determined for the tensile strength R_m

	AlSiCuMgNi	AlSiCuMgNi	AlSiCuMgNi	AlSiCuMgNi	AlSiCuMgNi	AlSiCuMgNi	AlSiCuMgNi	AlSiCuMgNi
AlSiCuMgNi - alloy							**Modification**	
Rm			**Modification**		**Heat treatment**		**Heat treatment**	
[MPa]	**Sand mould**	**Metal mould**	**Sand mould**	**Metal mould**	**Sand mould**	**Metal mould**	**Sand mould**	**Metal mould**
Mean	16,38	188,33	195,38	215,92	192,5	207,57	221,75	235,26
Weibull modulus	**59,71**	**68,03**	**69,83**	**82,58**	**67,84**	**70,78**	**105,79**	**116,22**
Sigma0	178	190	197	217,8	193	209	222	236
SD	3,75	3,179	3,785	3,044	3,846	4,153	2,847	2,565
Variability index	**0,2289**	**0,0169**	**0,0194**	**0,0141**	**0,0200**	**0,0200**	**0,0128**	**0,0109**
Wsk 1	**15,92**	**21,40**	**18,45**	**27,13**	**17,64**	**17,04**	**37,16**	**45,31**
Wsk 2	**2,608**	**40,302**	**36,046**	**58,576**	**33,955**	**35,376**	**82,399**	**106,596**
Assumed	Wsk3	Wsk3	Wsk3	Wsk3	Wsk3	Wsk3	Wsk3	Wsk3
200	**-29,24**	**-2,50**	**-0,85**	**4,32**	**-1,32**	**1,29**	**8,08**	**15,98**

3. Summary and conclusions

Analysing the obtained results in terms of their practical applicability, the following example is given for consideration. It is demanded that, besides many other requrements imposed on an alloy assigned for constructional purposes, like modest price, low density, etc., it should also have the tensile strength of minimum 150 MPa. In this study, attention has mainly been focussed on the processes and not on the requirements demanded by the specifications, although equation (8) allows us to consider an assumed value of the examined property (e.g. in Tables 2 and 3 the value of the tensile strength R_m equal to 200 MPa has been adopted).

The modified Al-Si-Cu-Mg-Ni alloy cast into metal moulds, whose R_m histogram is shown in Figure 8, was compared with a structural material called "Reference Material" - whose histogram R_m is shown in Figure 9.

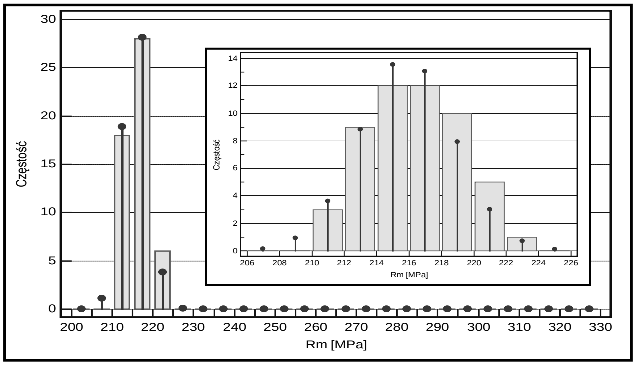

Fig. 8. Histogram of the tensile strength R_m of the modified Al-Si-Cu-Mg-Ni alloy cast into metal mould

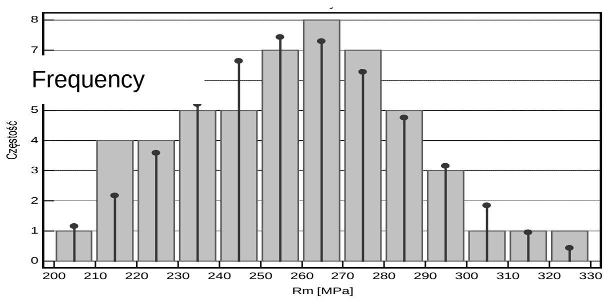

Fig. 9. Histogram of the tensile strength R_m of the reference material

In both cases, i.e. in the case of *silumin* and *reference material,* using Shapiro-Wilk test it has been stated that the distribution of the tensile strength R_m was not straying from the normal distribution (p=0,1466 and p=0,7082, respectively).

Figure 10 shows plotted diagrams of the survival abilities of the silumin and reference material.

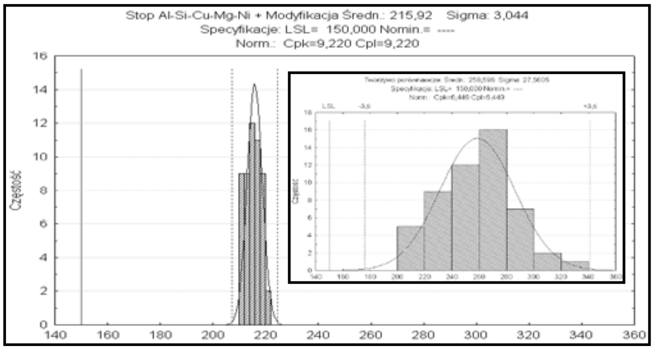

Fig. 10. Plotted diagrams of the R_m survival abilities of the *silumin* and *reference material*

From data presented in Figure 15 it follows that both alloys satisfy the requirements imposed by the specification; there are no values of R_m lower than 150 MPa. For silumin the survival ability index (C_{pk}) has a value equal to 9,22, for a reference material this value amounts to 6,45. Both these values are larger than the unity, and as such do not allow rejecting one of the alloys on account of the insufficient value of C_{pk}. So, other parameters should be taken into consideration. The reference material has an average R_m value of about 259 [MPa], while silumin offers the value of about 216 [MPa], which means that it is much lower (Table 3).

Table 3.
Reliability indices of silumin and reference material

Rm [MPa]	AlSiCuMgNi alloy + modification Metal mould	Reference material
Mean	215,92	258,60
Weibull modulus	**82,58**	**11,77**
Sigma0	217,8	272.3
SD	3,044	27,561
Variability index	**0,0141**	**0,1066**
Wsk 1	**27,13**	**0,43**
Wsk 2	**58,576**	**1,104**
Assumed	Wsk3	Wsk3
150	**17,88**	**0,46**

The reference material may be worth paying some attention since the prevailing percent share of samples made from it (about 87%) have the value of R_m higher than the samples cast from silumin characterised by maximum strength. On the other hand, compared with reference material, silumin offers smaller scatter of results. In quality analysis, it is the non-refutable principle that the narrower is the distribution range, the better is the material, because the most natural and fundamental index of process ability is the probability of manufacturing an element which will not be capable of satisfying the specification. The greater is this probability, the less reliable is the process. In this way one can reach a relationship between the material reliability and its ability of survival. If the reliability cannot be ensured, the estimation of the ability of survival will be of little value. This is illustrated by the estimated values of Weibull modulus and by the indices proposed by the authors and calculated from this modulus. The value of Weibull modulus allowing for the probability of survival of the examined alloys in respect of Rm is for the reference material about eight times smaller that it is for the silumin. On the other hand, the index interrelating Weibull modulus with the assumed value of the examined property and with the variability index (*wsk3* - equation 8) is for the silumin almost 40 times higher that it is for the reference material.

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